Estimating the natural rate of unemployment: Developing estimates from forecasting inflation

Abstract
The natural rate of unemployment is a fundamental concept in macroeconomics. The natural rate is widely used by policymakers to help determine fiscal and monetary policy. However, the natural rate presents a challenge in that one cannot directly observe the natural rate in the same manner that one can observe the unemployment rate. This challenge also makes it difficult to determine how accurate one’s estimates of the natural rate are. In our approach to estimate the natural rate, we employ various univariate smoothers and filters in order to extract the underlying trend from the cyclical unemployment rate. We also use a state-space model and the Kalman Filter along with an EM Algorithm to extract the unobserved state of the natural rate. We expand upon current methods used to estimate the natural rate by utilizing a more general multivariate autoregressive state-space model (MARSS) that incorporates structural changes in the labor market. When assessing the predictive ability of our estimates of the natural rate using the Phillips curve, we find that our estimates perform as well or better than those provided by the Congressional Budget Office.

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1 We acknowledge support from the NSF REU program at Lafayette College summer 2014. All remaining errors are ours.
2 Corresponding author.
Introduction

The natural rate of unemployment is an important statistic for policy makers to know; however, it cannot be observed in the economy. Since monetary policy makers especially rely on the unemployment gap to gauge the amount of economic slack in the economy having a timely and good estimate of the natural rate is important. Any forecast of the natural rate cannot be tested in the way a traditional forecast can be by comparing it to the actual realization since we never observe the actual. Instead, our analysis relies upon estimating the natural rate from both univariate and multivariate techniques and then taking the estimates of the natural rate and using them to forecast future inflation in a Phillips curve. This paper answers one main question: how useful is the estimate of the natural rate in forecasting future inflation? We answer this question by examining rolling forecasts.

The estimation techniques for the natural rate include both univariate and multivariate techniques. In the univariate framework, we use a Box Kernel smoother and the Hodrick-Prescott filter\(^3\). We also use the Kalman filter with control variables. The structural models include calculating at time-varying natural rate similar to Ball (2009), Gordon (1997) and Staiger, Stock and Watson (1997). Finally, we use multivariate autoregressive state space models (MARSS). As a benchmark, we use the CBO estimate of the short-term the natural rate including real-time data when possible. Our findings indicate that our estimates perform either better than or similar to the CBO estimate.

Related Literature

Milton Friedman (1968) and Edmund Phelps (1968) first developed the notion of the natural rate of unemployment in their work on the Phillips Curve during the 1960’s. Since then, economists have been plagued with the difficulty of estimating the natural rate and various

\(^3\) Previous versions of this paper have used kernel smoother and Butterworth filter.
approaches have been proposed. At first, economists just used a constant estimate of approximately five or six percent for the natural rate (Gordon 1981) and many businesses and some papers still make this assumption for simplicity sake and the lack of a consensus on a better approach. After experiencing periods of inflation despite levels of unemployment greater than five percent, economists began re-evaluating their assumptions about the natural rate. The Perry-weighted unemployment rate was proposed by George Perry (1970) as an attempt to improve the understanding of labor market slack, however, this weighting method has been used less in more recent years in favor of other techniques.

Over the past several decades, Robert Gordon has been a strong proponent of the Phillips curve, specifically his Triangle Model, even when Robert Lucas and Thomas Sargent (1978) were calling the Phillips curve an “econometric failure on a grand scale.” Within his model, Gordon utilizes the natural rate and thus, has significant interest in the estimation of the natural rate. Gordon (1997) argues for the estimation of a time-varying non-accelerating inflation rate of unemployment (TV-NAIRU), which King, Stock, and Watson (1995) and Staiger, Stock, and Watson (1997) have spent much time developing. This involves using a state-space model and an EM-algorithm to “back out” the implied the natural rate from the Phillips curve, given a reasonable assumption about the variance of the natural rate. This method has become the closest thing there is to a consensus within the literature.

Other methods include a method similar to the TV-NAIRU, but use a simple expression of the Phillips curve that does not include lags or supply shocks and does not require as sophisticated statistical methods. This method was first proposed by Ball and Mankiw (2002) and updated by Ball in (2009). The smoothing capability of the Kalman Filter has been another method employed to estimate the natural rate focusing on finding the trends of the job-separation
and job-finding rate individually (Tasci and Zaman 2010). Univariate smoothers are another method that can be employed, though they are not prevalent in the literature.

There are several important distinctions about estimating the natural rate. First, one must always specify a priori what they think the variability of the natural rate to be. Since the natural rate is unobserved, identifiability issues prevent one from estimating the variance of the natural rate. No method will resolve this issue, whether univariate or multivariate. Second, there has been a considerable discussion on the policy usefulness of the natural rate estimates because of their large confidence intervals (Staiger, Stock, and Watson 1997). This is an issue that still needs to be resolved. Instead of utilizing in-sample techniques, such as conventional confidence intervals, to evaluate the policy-usefulness of the natural rate estimates, we propose an out-of-sample method that compares the various natural rate estimates ability to predict or forecast inflation within the context of the Phillips curve. Some, such as Atkeson and Ohanian (2001) have questioned the usefulness of Phillips curve inflation forecasts as they have failed to beat naïve inflation estimates. Stock and Watson (2008) have re-evaluated the usefulness of Phillips curve inflation forecasts and conclude that naïve inflation estimates perform well when the unemployment rate is close to the natural rate, but when the labor market slack is large, utilizing the additional information in the Phillips curve is helpful. We continue with our use of the Phillips curve while acknowledging this issue.

The recognition that the natural rate is likely not a constant, along with the disparity between the natural rate in the United States and Europe, has created interest on what the level of the natural rate and what causes it to change. One argument is that there is hysteresis; hysteresis hypothesizes an interaction between the actual unemployment rate and the natural rate (Ball 1999, 2009). Recent research by Ghayad (2013) and Kroft et al. (2012) demonstrate a distinct
turning point at 26 weeks in the employability of the unemployed in the United States due to a stigma against the long-term unemployed. This corresponds to the point at which unemployment benefits typically expire. This stigma is one explanation for the proposed hysteresis. Other factors, such as reallocation of the labor force between industries, technological advancement that makes certain skills obsolete, changes in labor market laws, and demographic changes have all been reasons cited for changes in the natural rate.

Data

Since our focus is on estimating the natural rate we base most of the analysis on the U-3 unemployment rate from the Bureau of Labor Statistics. We also use various variables that attempt to incorporate structural changes in the labor market. One is the federal minimum wage, which is converted into real terms by deflating by the Consumer Price Index (CPI-U). Another variable is unemployment benefit extension dummy variable, which takes the value of one when specific legislation was passed to allow unemployment benefits to have extended duration. We also use unemployment benefit length, which is simply the maximum number of weeks that unemployment benefits are available for a given month. We also look at long-term unemployment rate, which is the percent of the labor force that is unemployed for longer than 26 weeks. We use 26 weeks due to the evidence that there is a distinct negative turning point in re-employability of the unemployed once they have been unemployed for longer than 26 weeks.

When we switch to the Phillips Curve, we use CPI-U as the inflation rate from the Bureau of Labor Statistics. In addition, there is a long literature that suggests incorporating other

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4 All rates, besides unemployment rates, are compound annual rate of changes: \( \left( \frac{x_t}{x_{t-1}} \right)^{\frac{\text{# of obs.}}{\text{year}}} - 1 \) * 100
5 From US DOL Wage and Hour Division [http://www.dol.gov/whd/minwage/chart.htm](http://www.dol.gov/whd/minwage/chart.htm)
8 Kroft et al. (2012) and Ghayad (2013) give an explanation of this phenomenon.
variables into the Phillips curve to enhance its fit. Therefore, we add a supply shock variable, Nixon price controls, import prices, productivity growth which are calculated as described below. Our supply shock variables are based on Gordon (1997). For the relative difference in food energy prices, we took the difference of the CPI rate of inflation minus the CPI less food and energy rate of inflation. The Nixon “on” and “off” variables help account for the Nixon price controls in the early 1970’s and are formulated the same as in Gordon (1982), except converted into a monthly series by maintaining the same value for all months in a given quarter. For the relative difference in import prices, we use a natural interpolating spline to convert the quarterly series for the level of the Bureau of Economic Analysis’ implicit price deflator for imports of goods and series into a monthly series. We then convert it into a rate and subtract rate of inflation measured from the GDP Deflator. For productivity deviation, we fit a piecewise linear trend function on the rate of change in the quarterly actual nonfarm private output per hour. As this trend is linear, it is easily converted into a monthly series. We then use a natural interpolating spline to turn the original quarterly nonfarm private output per hour series into a monthly series. We then subtract the linear trend from this monthly series to get the monthly productivity deviation.

Finally as our benchmark, we use the Congressional Budget Office’s estimate. The CBO publishes an estimate of both the short-term and the long-term the natural rate. We use the short-term estimate as our benchmark comparison as this is the series that the CBO uses for its inflation forecasts. Additionally, we use the real-time estimate of the natural rate from the CBO.

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The real-time data are available from 1991-2015 and before 1991 we use the short-term natural rate.\textsuperscript{10} Again, we use a natural spline to interpolate this series into a monthly series.

\textbf{Method of Evaluating the natural rate using the Phillips Curve}

We use one method to evaluate our estimates of the natural rate within the structure of the Phillips Curve. We compare the estimated inflation to actual inflation to measure the accuracy of the natural rate estimates. We also use rolling windows (20 year) in estimating the Phillips curve; the rolling window helps account for structural changes within the model.

For the univariate and Kalman filter techniques, we re-estimate both the natural rate and the Phillips Curve coefficients for each window\textsuperscript{11}. Notice that the CBO has an advantage over these methods as it incorporates future knowledge into its estimates of the natural rate until we are able to use real-time data. For the other structural models, we only estimate the natural rate once for the entire historical data series due to convergence issues and to minimize poor starting performance, but still estimate the coefficients on the Phillips curve for each window, which is equivalent to the CBO’s method of estimating the natural rate.

To answer the question of how useful is the estimate of the natural rate in forecasting future inflation, we employ $h$-period ahead forecasts within the Phillips Curve and estimate using a rolling window of data. Consistent with inflation forecasting literature, we estimate the following equation

$$\pi_{t+h,t} = \alpha(L)\pi_t + b(L)(U_t - U_t^N) + c(L)z_t + \epsilon_t$$

where

\textsuperscript{10} The short-term and long-term natural rates differ between the second quarter of 2008 and the third quarter of 2014.

\textsuperscript{11} Since the Kalman filter is a left-sided smoothing technique, we only estimate NAIRU once for the entire historical data series, which essentially equivalent to estimating it for each window, but helps avoid some of the poor starting performance of the Kalman filter.
\[ \pi_{t+h,t} = \left( \frac{\pi_{t+h}}{\pi_t} \right)^{12/h} - 1 \right) \times 100 \]

We use \( h \) values of 1, 3, 6, 9, 12, and 24 months for these rolling window forecasts.

**Lag Structure**

Our lag structure of the Phillips Curve deviates from Gordon (1997) as we utilize monthly data as opposed to quarterly data. To determine the lag structure of the Phillips Curve we look at the AIC/BIC for the dynamic simulation version of the Phillips Curve\(^{12}\) over our entire data series to guide us in our model specification. We use the CBO’s short-term estimate of the natural rate in our model for these tests. This gives the CBO a small advantage in our MSE testing as the model is built based upon their estimates of the natural rate. Table A indicates the lag structure preferred based on either AIC or BIC and the one we used in our models.

**Table A:**

<table>
<thead>
<tr>
<th>Variables</th>
<th>Minimum AIC Lag Structure</th>
<th>Minimum BIC Lag Structure</th>
<th>Lag Structure Chosen</th>
</tr>
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<tr>
<td>Inflation</td>
<td>61*</td>
<td>17</td>
<td>17</td>
</tr>
<tr>
<td>Unemployment Gap</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Food/Energy Prices</td>
<td>9</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Import Prices</td>
<td>2</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Productivity Deviation</td>
<td>0*</td>
<td>0*</td>
<td>2</td>
</tr>
<tr>
<td>Nixon “on”</td>
<td>0**</td>
<td>0**</td>
<td>0**</td>
</tr>
<tr>
<td>Nixon “off”</td>
<td>0**</td>
<td>0**</td>
<td>0**</td>
</tr>
</tbody>
</table>

*Minimum or maximum number of lags tested. **Constrained to just the contemporary value per (Gordon 1997).

Even though the AIC and BIC indicate no lags for the productivity deviation supply shock, in the rolling forecasts, productivity deviation is significant with a lag structure of 2.\(^{13}\) To be consistent with our lag structure throughout, for sake of simplicity, we specify the model as indicated in the table above for all Phillips Curves, unless explicitly indicated otherwise.\(^{14}\)

**Models to estimate the natural rate**

\(^{12}\) The dynamic simulation comes from Gordon’s triangle model.

\(^{13}\) Coefficient significance results are available upon request.

\(^{14}\) Note “Simple” Phillips Curve denotes a Phillips Curve with only lags of inflation and the unemployment gap.
Univariate Smoothers

The parameters chosen for each of the univariate methods is a “smoothness criterion” that is based upon our assumptions about the variability of the natural rate.

Kernel Smoother:

One of the simplest smoothers that can be used to estimate a trend in data is a kernel smoother. The estimated trend is calculated using a weighted average of the data. The general formula for finding the estimate of the smoothed function \( \hat{m}(x) \) using a kernel smoother is:

\[
\hat{m}_h(x) = \frac{\sum_{i=1}^{n} K_h(x - x_i)y_i}{\sum_{i=1}^{n} K_h(x - x_i)}
\]

Here the function \( K(\cdot) \) is the kernel which determines how the different data points are weighted. Examples of typical kernels are the box kernel, which takes the average of all data points within a certain window of the \( x \)-value for which we are estimating the underlying function, and the normal kernel, which weights data points further from the \( x \)-value of interest less than points closer to it with the weights determined by placing a normal curve centered on the \( x \)-value of interest. The value \( h \) represents the bandwidth of the kernel which essentially dictates how many data points are given weight when estimating the function for a given \( x \)-value.

For the box kernel, it is the width of the window; for the normal kernel, it is related to the standard deviation of the normal density being used as the weight. We tested a variety of kernels and choices of bandwidth. For simplicity, we report the results from the simple box kernel, often referred to as simply a moving average.

HP Filter:

The Hodrick-Prescott (or HP) filter is a band pass filter commonly used in macroeconomics. The goal of this filter is to estimate the gradual trends in the data while removing the high frequency noise. Here, we work under the assumption that the series of data
points are comprised of a true function plus the addition of normally distributed noise; thus, 
\( y_t = m_t + \varepsilon_t \). To find an estimate of \( m_t \), one typical criterion is to find the function \( \hat{m}_t \) that minimizes the sum of squares error \( (y_t - \hat{m}_t)^2 \). However, in a non-parametric setting, we could simply set our estimate to be the actual data points, resulting in a sum of squares error of 0 that is simply a wiggly fit that interpolates the data. To avoid this issue, it is desired to impose a penalty to the “wiggliness” of the fit. The object is to find a function that balances a low sum of squares error with this degree of wiggliness. The solution is to find the function \( m_t \) that minimizes the loss function:

\[
\left\{ \sum_{t=1}^{T} (y_t - m_t)^2 + \lambda \sum_{t=2}^{T-1} (m_{t-1} - 2m_t + m_{t+1})^2 \right\}
\]

The function that minimizes the loss function is our estimate \( \hat{m}_t \) for the general trend of the data. The value \( \lambda \) determines the penalty that is imposed on the wiggliness of the fit. Note that a value of 0 imposes no penalty while an infinite value requires the fit to be a straight line. Different choices of \( \lambda \) will lead to different fits. Hodrick and Prescott (1997) suggested a value of 1600 for quarterly data. Ravn and Uhlig (2002) argued that \( \lambda \) should be 6.25 for annual data and 129600 for monthly data, the value typically used in analysis of macroeconomics data. This value gives satisfactory mean squared error in the rolling forecasts, though we tested different \( \lambda \) terms for the HP filter.

**The Kalman Filter:**

The Kalman Filter is a recursive filtering process defining a state variable \( x_t \) and observed variable \( z_t \).

\[
x_t = Ax_{t-1} + BU_t + w_{t-1}
\]
\[
z_t = Hx_t + v_t
\]
The state variable \( x_t \) represents the unobservable component returning the estimate of the natural rate. The observed variable, \( z_t \), represents the actual unemployment rate. The structure of the model is incorporated into matrix \( A \) relating the state at time \( t - 1 \) to the current state \( t \). The addition of \( BU_t \) allows for the contribution of control variable(s) that potentially explain changes in the unobservable component itself. These contributions explain components left unexplained by the actual observations made. In the case of the natural rate, we consider variables such as real minimum wage, unemployment benefits and long-term unemployment to explain the natural rate in addition to the contributions by changes in the observed unemployment rate. The process noise, \( w_{t-1} \), allows for variation in the structural process itself. The matrix \( H \) relates the state in time \( t \) to the observed measurement, \( z_t \). The measurement noise, \( v_t \), accounts for the variance in the actual data itself.

The Kalman Filter performs an iterative process of time and measurement updates to determine the state variable \( x_t \). The time update predicts the state variable by projecting forward the current state allowing for specified process error. Then, the measurement update corrects this prediction by incorporating the new observed measurement.

We also include control variables in the value of \( U_t \), which contribute to fluctuations in the natural rate. Minimum wage (in real terms), extended unemployment benefits and long term unemployment are tested in the control variable term. Control variables were tested alone, pairwise and all three jointly. For baseline comparisons, control variables were eliminated.

**Structural Models**

\[
W \sim N(0, Q) \\
V \sim N(0, R)
\]
Besides the univariate smoothers/filters and the Kalman filter, there are also structural and multivariate models that can be used to estimate the natural rate. These involve backing out the implied natural rate from the Phillips Curve.

**Gordon/Staiger, Stock, & Watson TV-NAIRU:**

King et al. (1995), Gordon (1997), and Staiger et al. (1997) back out the natural rate from the full Phillips Curve. This method assumes that the natural rate follows a random walk and has the structure of

\[
\pi_t = \alpha(L)\pi_{t-1} + b(L)(U_t - U_t^N) + c(L)\varepsilon_t + \varepsilon_t
\]

and

\[
U_t^N = U_{t-1}^N + \varepsilon_t
\]

and the error terms are both assumed to be normally behaved. In order to avoid identifiability issues, at least one of the standard deviations of the error terms must be specified *a priori*. Specifying the standard deviation of the second equation makes the most sense, as Gordon (1997) notes this choice is logically equivalent to the smoothness criterion that is specified in the univariate methods.

This specification is a state-space model and can use the Kalman Filter along with the MLE and the EM algorithm to estimate both the standard deviation of the error term in the first equation and the natural rate.\textsuperscript{15}

**MARSS Model:**

In the above state-space model, the standard deviation of the second equation determines not only the volatility of the natural rate but also the standard confidence intervals on the estimate of the natural rate. Increasing the precision of the estimates of the natural rate is an important aspect of strengthening the concept of the natural rate as discussed in Staiger, Stock,

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\textsuperscript{15} References on this method include (Hamilton 1994), (Watson and Engle 1983), & (Shumway and Stoffer 1982).
and Watson (1997). However, one cannot simply decrease the standard deviation in order to
decrease the uncertainty of the natural rate estimate, as the calculated confidence intervals
assume that the given \textit{a priori} standard deviation is the true value. The standard deviation in the
second equation incorporates all the shocks that affect the natural rate, such as structural changes
in the labor market. By incorporating these structural changes directly into the second equation,
we can reduce the \textit{a priori} standard deviation and thus, potentially increase the precision of our
estimates of the natural rate. Furthermore, adding the labor market structure variables will also
increase the accuracy of the estimates as it reduces any potential bias. We can also allow the
natural rate to depart from its random walk assumption and even add additional lags of the
natural rate to the equation. Furthermore, this model allows us to more directly test the hysteresis
or long-term persistence theories of unemployment, such as (Ball 1999) and (Ball 2009).

Adding these model changes in the labor market to the state-space model, we get a
general model of

$$
\pi_t = \alpha(L)\pi_{t-1} + b(L)(U_t - U_t^N) + c(L)z_t + \epsilon_t
$$

and

$$
U_t^N = \delta(L)U_{t-1}^N + \gamma(L)D_t + \epsilon_t
$$

where $D_t$ contains variables address structural changes within the labor market. We look at
minimum wage, unemployment benefit extensions, maximum unemployment benefit duration,
and long-term unemployment. This family of models is known as multivariate autoregressive
state-space models or MARSS. The Gordon/Staiger, Stock, and Watson TV-NAIRU method is
actually a member of the more general MARSS family.

As far as we know, the use of this MARSS formulation, where the $\gamma(L)D_t$ term is
included in the state equation, is at least absent from the economic literature examining the
natural rate and largely absent from economic literature in general. This is likely due to the fact that the derivation of the EM algorithm for this model has only been recently made available by Holmes 2014.

The MARSS model with $U_t^N$ modeled as an AR(1) process with the contemporary value of various labor market features does perform better than the TV-NAIRU results with the corresponding Q value in dynamic simulations. This is especially true at lower Q values, which is what we would expect. The Q value determines how much variance is allowed within the state equation. At higher Q values, the state-space model naturally incorporates these structural labor market changes, or labor market shocks, into the estimate of $U_t^N$. High Q values also allow our estimate of $U_t^N$ to soak up other shocks that affect inflation but may not actually change the true value of $U_t^N$. At lower Q values, the state-space model is restricted from incorporating as much of the labor market shocks and other shocks that affect inflation that it may soak up at higher Q values. Therefore, by directly modelling the labor market shocks in the state equation, we are able to reduce our Q value, which should increase the precision and accuracy of our estimates as we are directly modelling the labor market shocks and avoiding soaking up other shocks that affect inflation but not necessarily the true value of $U_t^N$. It furthermore allows for a more concrete understanding of how $U_t^N$ moves as a result of various labor market changes, as we can directly test whether a labor market variable is statistically significant in determining $U_t^N$.  

**Discussion of Results**

The result appear in Tables 1 to 4. Overall, the models that we use to estimate the natural rate are better than the CBO estimate. We generate 126 inflation forecasts and 63% (79 instances) are significantly better at the 5% level than the inflation forecast from the CBO’s

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16 Appendix A contains an example of MARSS method and an explanation of the method.
natural rate. At the 10% significance level there are an additional 10 instances where our models are significantly better than the CBO’s. The majority of the significant results are when we estimate a TV-NAIRU following the procedures of Gordon and the MARSS model.

There are some differences in the performance of the models. The univariate models (no matter the method) are statistically indistinguishable from the CBO estimate except at the 2 shortest time horizons (1-month and 3-month). The Kalman filter with controls (most importantly, controlling for the real minimum wage) is significantly better than the CBO estimate at the 1, 3, 6 month time horizons. Figures 1,2 and 3 show some of the estimates of the natural rate from the Kalman filter against actual unemployment.17

The Gordon and Staiger, Stock and Watson methods yield many significant results. Under the time-varying method with rolling forecasts, we find significant differences at the 1, 3, 6, 9 and 12-month time horizons with all smoothing parameters in the Phillips curve which includes supply shocks, productivity shocks and import prices18. Even when estimating a simpler Phillips curve that only includes lags of inflation and the unemployment gap our estimate of the natural rate produces better forecasts of inflation than the CBO’s estimate in the majority of cases. These results suggest that incorporating more information does help forecast inflation and adds information to the natural rate estimate we produce. Figure 4 shows our estimates of the natural rate against actual unemployment.

The results from forecasting inflation using the MARSS method to estimate the natural rate are statistically better than the CBO in most horizons. The model we have estimated only includes the minimum wage as a control variable. Because of the superiority of the TV-NAIRU

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17 When using the Kalman filter we tried a wide variety of models, control variables and smoothing parameters. We choose to present some of the better fitting models but the remainder of results is available upon request.

18 These Phillips curve models are denoted in the tables as “normal” as opposed to the “simple” models that exclude supply shocks, productivity shocks and import prices.
and the MARSS models, we test if those two sets of models are significantly different from each other. We find that they appear to be equivalent. Figure 5 provide visual confirmation that some of the models may be useful for understanding the amount of slack in the economy. This is preliminary evidence that there are models that could provide real-time estimates of the natural rate. A more intensive investigation of the MARSS models needs to be conducted.

The benefits of the approach that we have taken even when statistically equivalent to the CBO estimate is that our estimate is available in real-time and can be updated every month. The CBO updates the estimate of the natural rate at the beginning of every calendar year; therefore, our models can provide an update of the natural rate when economic conditions are changing over the course of a year such as during the Great Recession.

**Conclusion**

The natural rate of unemployment is not observed in the economy. Monetary policy makers need a timely and accurate estimate of the natural rate in order to make policy decisions. We develop many estimates of the natural rate from several different methods and test our estimates of the natural rate by using them to forecast future inflation in a Phillips curve.

We use the CBO estimate of the short-term natural rate as our benchmark and we use the real-time data after 1990. We find that our estimates perform either better than or similar to the CBO estimate especially the MARSS estimates. Future research should work to explore this class of models to determine better estimates of the natural rate and more importantly, to determine which additional control variables help to estimate the natural rate the most.
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Table 1: Univariate Methods Rolling Forecasts Results:

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<th>Dates</th>
<th>Method &amp; Parameter</th>
<th>Phillips Curve</th>
<th>h=1</th>
<th>h=3</th>
<th>h=6</th>
<th>h=9</th>
<th>h=12</th>
<th>h=24</th>
</tr>
</thead>
<tbody>
<tr>
<td>1977/6 – 2012/05</td>
<td>CBO real-time</td>
<td>Normal</td>
<td>10.619</td>
<td>4.197</td>
<td>3.004</td>
<td>2.819</td>
<td>2.836</td>
<td>3.610</td>
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<td>1977/6 – 2012/05</td>
<td>Box Kernel 60</td>
<td>Normal</td>
<td>8.420** (.&lt;.001)</td>
<td>3.654* (0.020)</td>
<td>2.748 (0.186)</td>
<td>2.589 (0.151)</td>
<td>2.623 (0.128)</td>
<td>3.486 (0.189)</td>
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<tr>
<td>1977/6 – 2012/05</td>
<td>HP Filter 129600</td>
<td>Normal</td>
<td>8.595* (&lt;.001)</td>
<td>3.762* (0.031)</td>
<td>2.872 (0.258)</td>
<td>2.686 (0.201)</td>
<td>2.697 (0.112)</td>
<td>3.389 (0.056)</td>
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Table 2: Kalman Filter Rolling Forecast Results

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<tr>
<th>Dates</th>
<th>Method</th>
<th>Phillips Curve</th>
<th>Window</th>
<th>h=1</th>
<th>h=3</th>
<th>h=6</th>
<th>h=9</th>
<th>h=12</th>
<th>h=24</th>
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<tbody>
<tr>
<td>1977/6 – 2012/5</td>
<td>CBO Real-time</td>
<td>Normal</td>
<td>240 months</td>
<td>10.619</td>
<td>4.197</td>
<td>3.004</td>
<td>2.819</td>
<td>2.836</td>
<td>3.610</td>
</tr>
<tr>
<td>1977/6 – 2012/5</td>
<td>No Control Random Walk</td>
<td>Normal</td>
<td>240 months</td>
<td>9.421** (&lt;.001)</td>
<td>3.838* (.041)</td>
<td>2.869 (.139)</td>
<td>2.761 (.276)</td>
<td>2.805 (.385)</td>
<td>3.642 (.745)</td>
</tr>
<tr>
<td>1977/6 – 2012/5</td>
<td>Control: Minimum Wage</td>
<td>Normal</td>
<td>240 months</td>
<td>3.991* (.000)</td>
<td>2.309* (.000)</td>
<td>2.284* (.027)</td>
<td>2.419 (.069)</td>
<td>2.561 (.126)</td>
<td>3.480 (.548)</td>
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<tr>
<td>1977/6 – 2012/5</td>
<td>Control: Minimum Wage and Unemployment Benefits</td>
<td>Normal</td>
<td>240 Months</td>
<td>3.678* (&lt;.001)</td>
<td>2.403* (&lt;.001)</td>
<td>2.450* (.022)</td>
<td>2.588 (.079)</td>
<td>2.729 (.245)</td>
<td>3.762 (.622)</td>
</tr>
<tr>
<td>1977/6 – 2012/5</td>
<td>No control Random Walk</td>
<td>Simple</td>
<td>240 months</td>
<td>10.008* (&lt;.001)</td>
<td>5.710 (.067)</td>
<td>4.172 (.277)</td>
<td>3.488 (.476)</td>
<td>3.332 (.650)</td>
<td>3.975 (.790)</td>
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</tbody>
</table>

*MSE for Rolling Forecast. *Diebold-Mariano one-sided P-value (our estimate’s error<CBO’s) with loss function power equal to 2.
* significant at 5% level
Table 3: TV-NAIRU Rolling Forecast (See Figure 4)

<table>
<thead>
<tr>
<th>Dates</th>
<th>Method</th>
<th>Phillips Curve</th>
<th>Window</th>
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<td>3.004</td>
<td>2.819</td>
<td>2.836</td>
<td>3.610</td>
</tr>
<tr>
<td>1977/6-2012/5</td>
<td>TV-NAIRU (Q=0.002)</td>
<td>Normal</td>
<td>240 months</td>
<td>9.189* (0.001)</td>
<td>3.278* (0.004)</td>
<td>2.520* (0.016)</td>
<td>2.495* (0.016)</td>
<td>2.492* (0.013)</td>
<td>3.298 (0.130)</td>
</tr>
<tr>
<td>1977/6-2012/5</td>
<td>TV-NAIRU (Q=0.001)</td>
<td>Normal</td>
<td>240 months</td>
<td>9.208* (0.001)</td>
<td>3.493* (0.005)</td>
<td>2.634* (0.020)</td>
<td>2.585* (0.022)</td>
<td>2.604* (0.013)</td>
<td>3.410 (0.102)</td>
</tr>
<tr>
<td>1977/6-2012/5</td>
<td>TV-NAIRU (Q=0.0005)</td>
<td>Normal</td>
<td>240 months</td>
<td>9.111* (0.001)</td>
<td>3.582* (0.003)</td>
<td>2.701* (0.017)</td>
<td>2.643* (0.023)</td>
<td>2.681* (0.016)</td>
<td>3.495 (0.059)</td>
</tr>
<tr>
<td>1977/6-2012/5</td>
<td>TV-NAIRU (Q=0.00025)</td>
<td>Normal</td>
<td>240 months</td>
<td>9.029* (0.001)</td>
<td>3.613* (0.001)</td>
<td>2.730* (0.013)</td>
<td>2.668* (0.023)</td>
<td>2.717* (0.022)</td>
<td>3.534 (0.051)</td>
</tr>
<tr>
<td>1977/6-2012/5</td>
<td>TV-NAIRU (Q=0.00005)</td>
<td>Normal</td>
<td>240 months</td>
<td>8.881* (0.001)</td>
<td>3.610* (0.001)</td>
<td>2.741* (0.010)</td>
<td>2.677* (0.025)</td>
<td>2.736* (0.042)</td>
<td>3.553 (0.200)</td>
</tr>
<tr>
<td>1977/6-2012/5</td>
<td>TV-NAIRU (Q=0.002)</td>
<td>Simple</td>
<td>240 months</td>
<td>9.850 (0.001)</td>
<td>5.118* (0.004)</td>
<td>3.824* (0.028)</td>
<td>3.205* (0.041)</td>
<td>3.005* (0.041)</td>
<td>3.665 (0.191)</td>
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<tr>
<td>1977/6-2012/5</td>
<td>TV-NAIRU (Q=0.001)</td>
<td>Simple</td>
<td>240 months</td>
<td>9.869* (0.001)</td>
<td>5.371* (0.006)</td>
<td>3.938* (0.031)</td>
<td>3.300 (0.056)</td>
<td>3.114 (0.066)</td>
<td>3.745 (0.186)</td>
</tr>
<tr>
<td>1977/6-2012/5</td>
<td>TV-NAIRU (Q=0.0005)</td>
<td>Simple</td>
<td>240 months</td>
<td>9.752* (0.001)</td>
<td>5.477* (0.003)</td>
<td>4.007* (0.031)</td>
<td>3.361 (0.064)</td>
<td>3.190 (0.086)</td>
<td>3.821 (0.162)</td>
</tr>
<tr>
<td>1977/6-2012/5</td>
<td>TV-NAIRU (Q=0.00025)</td>
<td>Simple</td>
<td>240 months</td>
<td>9.653* (0.001)</td>
<td>5.514* (0.002)</td>
<td>4.039* (0.031)</td>
<td>3.390 (0.075)</td>
<td>3.229 (0.113)</td>
<td>3.863 (0.151)</td>
</tr>
<tr>
<td>1977/6-2012/5</td>
<td>TV-NAIRU (Q=0.00005)</td>
<td>Simple</td>
<td>240 months</td>
<td>9.497* (0.001)</td>
<td>5.514* (0.001)</td>
<td>4.052* (0.037)</td>
<td>3.405 (0.115)</td>
<td>3.253 (0.192)</td>
<td>3.890 (0.305)</td>
</tr>
</tbody>
</table>

1MSE for Rolling Forecast. 2Diebold-Mariano one-sided P-value (our estimate’s error<CBO’s) with loss function power equal to 2.
* significant at 5% level

Table 4: MARSS Rolling Forecast VERSUS CBO & TVNAIRU (See Figure 5)

<table>
<thead>
<tr>
<th>Dates</th>
<th>Method</th>
<th>Phillips Curve</th>
<th>Window</th>
<th>h=1</th>
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<tr>
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<td>Normal</td>
<td>240 months</td>
<td>10.619</td>
<td>4.197</td>
<td>3.004</td>
<td>2.819</td>
<td>2.836</td>
<td>3.610</td>
</tr>
<tr>
<td>1977/6-2012/5</td>
<td>MARSS (AR(1), Min Wage (Q=0.002))</td>
<td>Normal</td>
<td>240 months</td>
<td>9.370* (0.001)</td>
<td>3.361* (0.001)</td>
<td>2.574* (0.004)</td>
<td>2.530* (0.004)</td>
<td>2.523* (0.005)</td>
<td>3.335 (0.142)</td>
</tr>
<tr>
<td>1977/6-2012/5</td>
<td>MARSS (AR(1), Min Wage (Q=0.001))</td>
<td>Normal</td>
<td>240 months</td>
<td>9.804* (0.001)</td>
<td>3.702* (0.005)</td>
<td>2.731* (0.013)</td>
<td>2.639* (0.016)</td>
<td>2.643* (0.006)</td>
<td>3.439 (0.136)</td>
</tr>
<tr>
<td>1977/6-2012/5</td>
<td>MARSS (AR(1), Min Wage (Q=0.0005))</td>
<td>Normal</td>
<td>240 months</td>
<td>9.927* (0.001)</td>
<td>3.857* (0.012)</td>
<td>2.824* (0.032)</td>
<td>2.712* (0.047)</td>
<td>2.729* (0.005)</td>
<td>3.533 (0.171)</td>
</tr>
<tr>
<td>1977/6-2012/5</td>
<td>MARSS (AR(1), Min Wage (Q=0.00025))</td>
<td>Normal</td>
<td>240 months</td>
<td>9.903* (0.001)</td>
<td>3.901* (0.011)</td>
<td>2.858* (0.035)</td>
<td>2.740* (0.096)</td>
<td>2.768* (0.047)</td>
<td>3.577 (0.265)</td>
</tr>
<tr>
<td>1977/6-2012/5</td>
<td>MARSS (AR(1), Min Wage (Q=0.00005))</td>
<td>Normal</td>
<td>240 months</td>
<td>9.351* (0.001)</td>
<td>3.758* (0.001)</td>
<td>2.804* (0.014)</td>
<td>2.713* (0.036)</td>
<td>2.762* (0.063)</td>
<td>3.575 (0.293)</td>
</tr>
</tbody>
</table>

1MSE for dynamic simulation. 2Diebold-Mariano one-sided P-value (our estimate's error<CBO’s) with loss function power equal to 2. 3Diebold-Mariano one-sided P-value (MARSS<TVNAIRU) with loss function power equal to 2.
* significant at 5% level comparing MARSS model to CBO real-time
Figure 1:
Kalman Filter – Random Walk, No Control, R=3, Q=.00005

Figure 2:
Kalman Filter – AR(2), Minimum Wage (1), R=2, Q=.001
Figure 3:
Kalman Filter – AR(2), Minimum Wage (1) & UB Extension (1), R=3, Q=.00005

Figure 4: TV-NAIRU (Q=.002 - Red, Q=.001 - Green, Q=.0005 - Blue, Q=.00025 – Light Blue, Q=.00005 - Magenta)
Figure 5: MARSS – AR(1), Min Wage (1), (Q=.002, Q=.001, Q=.0005, Q=.00025, Q=.00005)
Appendix A: Example of MARSS and Explanation of Method

Consider the MARSS model where the Phillips Curve contains two lags of inflation ($\pi_t$), the contemporary along with two lags of the unemployment gap ($U_t - U_t^N$), and the contemporary along with one lag of a “supply shock” variable ($z_t$). Let NAIRU ($U_t^N$) be specified as an AR(3) process with the contemporary and one lag of the real minimum wage ($w_t$). The state-space matrix representation of this model needed for the EM algorithm is below.

$$
\pi_t = [-b_0 - b_1 - b_2] \begin{bmatrix} 
U_t^N \\
U_{t-1}^N \\
U_{t-2}^N 
\end{bmatrix} + [\alpha_1 \alpha_2 b_0 b_1 b_2 c_0 c_1] + \epsilon_t
$$

and

$$
\begin{bmatrix} 
U_t^N \\
U_{t-1}^N \\
U_{t-2}^N 
\end{bmatrix} = \begin{bmatrix} 
\delta_1 & \delta_2 & \delta_3 \\
1 & 0 & 0 \\
0 & 1 & 0 
\end{bmatrix} \begin{bmatrix} 
U_{t-1}^N \\
U_{t-2}^N \\
U_{t-3}^N 
\end{bmatrix} + \begin{bmatrix} 
\gamma_0 \\
\gamma_1 \\
0 
\end{bmatrix} \begin{bmatrix} 
w_t \\
w_{t-1} 
\end{bmatrix} + \begin{bmatrix} 
e_t \\
0 
\end{bmatrix}
$$

$$
e_t \sim N(0, \sigma^2)
$$

and

$$
\begin{bmatrix} 
U_0^N \\
U_1^N \\
U_2^N 
\end{bmatrix} \sim MVN(\omega, \Lambda)
$$

The parameters $\alpha, b,$ and $c$ are estimated using OLS and the Phillips Curve with the mean of the unemployment rate serving as a proxy for NAIRU, while $\sigma^2$ is specified \textit{a priori} per the previous discussion in the TV-NAIRU section. Initial conditions for $\omega$ should also be given to start the EM algorithm, which we use the mean unemployment rate for the data series for. The parameters $\delta, \gamma, R, \omega, \Lambda$ along with $U_t^N$ are then estimated using the EM algorithm within the
Kalman Filter. Ideally, one would want to estimate all the parameters via the EM algorithm. However, due to identity issues, this is not possible. This overall model can easily be expanded to incorporate any specification that is a member of the MARSS family.