Asset Bubbles and Global Imbalances

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Abstract

We analyze the relationships between bubbles, capital flows and economic activities in a rational bubble model with two open economies. We show a reinforcing relationship between global imbalances and bubbles. Capital flows from South to North facilitate the emergence and the size of bubbles in the North. Bubbles in the North in turn facilitate South-to-North capital flows. The model can simultaneously explain several stylized features of recent bubble episodes.

Keywords: Rational bubbles · global imbalances · financial frictions · credit boom

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1 Introduction

The recent boom and bust of asset prices in the U.S. and the subsequent financial crisis have renewed the interest among economists and policymakers in understanding the relationships between capital flows, asset bubbles and boom-busts in economic activities. Specifically, there are three stylized features that characterize this boom-bust episode:

1. Global imbalances: Over the past few decades, capital has flown in large quantities from emerging economies to developed ones. In particular, the U.S. has been a net capital importer since the 1980s, especially with inflows from emerging economies in the years preceding the Great Recession. At its peak in 2006, the U.S. current account deficit exceeded $600 billion, or 6% of GDP. In contrast, emerging economies, especially China and other emerging Asian economies, have experienced expanding current account surpluses. This phenomenon of global imbalances and upstream capital flows, which coincides with a general decline in world interest rates, has been well documented (see, e.g., Bernanke 2005, Caballero et al. 2008, Mendoza et al. 2009, Gourinchas and Rey 2013).

2. Boom and bust in asset prices: The peak period, between 2002 and 2007, of capital flows from emerging economies into the U.S. was associated with a spectacular boom and bust in asset prices, especially housing prices. For instance, the S&P/Case-Shiller U.S. National Home Price Index rose by 85% between January 2000 and July 2006, before dropping 27% below its peak value by February 2012. Furthermore, it is difficult to explain much of these fluctuations by changes in economic fundamentals, such as demographics, construction costs, or interest rates (Case and Shiller 2003, Mian and Sufi 2014 and Shiller 2015). Furthermore, several prominent economists and policymakers have argued that the glut of savings flowing from emerging economies into the U.S. after the East Asian crisis might have caused or at least facilitated the boom in housing prices prior to the financial crisis (Bernanke et al. 2007, Greenspan 2009, Yellen 2009, Obstfeld and Rogoff 2009, Rajan 2011, Stiglitz 2012, Summers 2014).

3. Fluctuations in economic activities: The boom and bust in asset prices were associated with significant fluctuations in economic activities. The boom in housing prices in the 2000s was associated with a credit boom for both households and firms (Chaney et al.

\[1\] Using state-level data, Stewen and Hoffmann (2015) document that housing prices were more sensitive to capital inflows in states that had opened their banking markets to out-of-state banks earlier, providing some empirical evidence for this claim.
On the other hand, the collapse of housing prices in 2007 was associated with contractions in aggregate economic activities (e.g., Yellen 2009, Mian and Sufi 2010, 2014). The observations above are also consistent with broader empirical regularities about bubble episodes across countries: capital inflows and credit expansion during the boom phase, but sharp economic contractions and current account readjustment during the bust phase (Mendoza and Terrones 2008, 2012, Reinhart and Rogoff 2008, 2009 and Kindleberger and Aliber 2011). The experiences of Spain and Ireland in the 2000s follow this pattern: strong capital inflows, booms in real estate prices, a loosening of collateral constraints for households and firms, a boom in economic activities, and eventually a collapse in real estate prices that led to a sharp reduction in capital inflows, deleveraging by household and firms, and a severe recession (Lane 2011, Veld et al. 2014, Mian and Sufi 2014).

Motivated by these observations, we develop a tractable framework of asset bubbles, capital flows and economic activities. We generalize the rational bubble framework in a closed economy, as pioneered by Samuelson (1958), Diamond (1965), Tirole (1985), into a setting with two large open economies, called the North and the South. The North represents the U.S., while the South represents emerging economies such as China. Each economy consists of overlapping generations of agents who provide labor and capital.

We then introduce heterogeneous productivity and a credit friction. Agents have different levels of entrepreneurial productivity in producing capital, generating a natural motive for borrowing and lending in each economy. In the absence of financial frictions, the most productive agents would undertake all capital investment, while other agents would simply lend. This would lead to an equilibrium in which the interest rate would be equal to the return on capital investment made by the most productive agents. However, we assume that due to imperfections in the credit market, agents face a constraint on their ability to borrow. Because of this credit friction, the interest rate is determined by the return to capital investment made by marginal investors – agents who are indifferent between investing and lending.

Next, we introduce an asymmetry in financial development. To reflect the fact that emerging economies are less financially developed than the U.S., we assume that the credit friction is stronger in the South than in the North. Because of asymmetric frictions, the

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2 For instance, Chaney et al. (2012) found that between 1993 and 2007, a $1 increase in the value of the real estate that firms own, led to a $0.06 increase in investment by a representative U.S. corporation. Gan (2007) also provides evidence for a similar effect of the Japanese real estate bubble on debt capacities and investments of firms in Japan.
autarky interest rate is lower in the South than in the North. Then, as in the global imbalances literature, financial integration leads to upstream capital flows, as credit flows from the South to the North. As a consequence, financial integration lowers the interest rate in the North and raises it in the South. One interpretation of this result is that the gradual integration of emerging economies into the global financial market leads to a lowering of the world interest rate.

Finally, we introduce bubbles. As in the classic Samuelson-Diamond-Tirole (SDT hereafter) framework, bubbles are assets with no fundamental value but are traded at positive prices because agents expect to be able to resell them later. Bubbles can exist either in the North or in the South. As is well known, bubbles can naturally arise in economies with constraints against lending because they provide less productive agents an alternative storage of wealth, crowding out less productive capital investment and raising aggregate consumption. Furthermore, we follow Martin and Ventura (2012) and assume that agents can create new bubbles. Bubble creation can be interpreted, for instance, as entrepreneurs creating new firms or building new structures. New bubbles increase the net worth of agents, hence relaxing their credit constraints and allowing more productive agents to borrow more. Consequently, a boom in the values of bubbles leads to a boom in credit. Through this net worth effect, bubbles can crowd in capital investment and output, but their collapse leads to sharp economic contractions. As in most of the new generation of rational bubble models, including Caballero and Krishnamurthy (2006), Kocherlakota (2009), Miao and Wang (2011), Farhi and Tirole (2012), Martin and Ventura (2012) and Hirano and Yanagawa (2014), this crowd-in mechanism allows the model to be consistent with the fact that investment and output usually contract following the collapse of a bubble episode.

Our main results are as follows. First, financial integration facilitates the emergence of bubbles in the North. This is because financial integration causes capital flows from the South to the North due to the asymmetry in financial development. Capital inflows lower the interest rate in the North and hence facilitates the emergence of Northern bubbles. We also show that capital inflows raise the size of the bubble relative to the North’s economy. We interpret this result as a formalization of the claim mentioned above that the inflows of savings from developing countries contributed to a housing bubble in the U.S. Second, we

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3 Alternative ways of modeling expansionary bubbles include assuming that entrepreneurial activities that require funding arrive at a later period in an agent’s life (e.g., Caballero and Krishnamurthy 2006 and Farhi and Tirole 2012) or assuming that agents are infinite-lived and alternate between being in high and low productivity states (Hirano and Yanagawa 2014).

4 In contrast, the SDT framework predicts investment and output booms after a bubble burst.
show that the emergence of a bubble in the North raise the interest rate in the North. Thus, the bubble attracts further capital inflows to the North. Putting these results together, our theory predicts a reinforcing relationship between global imbalances and bubbles: capital flows from South to North facilitates bubbles in the North, and vice versa.

Our model also predicts a relationship between the boom and bust of a bubble episode and fluctuations in the aggregate economy. In the boom phase of a bubble episode, the North experiences expansions in aggregate investment, output, consumption, as well as an increase in the stock of debt and a deterioration of the current account. However, the collapse of the bubble precipitates contractions in aggregate economic activities in the North, including debt deleveraging. We interpret these predictions of the effects of large bubbles as consistent with the aforementioned stylized facts for the U.S.

Related literature: According to our knowledge, there has been a relative shortage of theoretical framework to systematically analyze the underlying relationships between asset bubbles, capital flows and fluctuations in economic activities. Our paper is most related to the literature on rational bubbles, which has a long heritage, dating back to the original model by Samuelson (1958) and later models by Diamond (1965), Tirole (1985) and Weil (1987). Much of the literature has focused on a closed economy setting. Recently, such papers include Kocherlakota (2009), Miao and Wang (2011), Martin and Ventura (2012), Farhi and Tirole (2012), Hirano and Yanagawa (2014), Aoki and Nikolov (2015), Ikeda and Phan (2016), Bengui and Phan (2017) and Hanson and Phan (2017). For a large open economy setting, however, there are only a few papers, including Kraay and Ventura (2007) and Basco (2013). While Kraay and Ventura (2007) focus on the effects of the dot-com bubble on the pattern of capital flows into the U.S. and Basco (2013) focuses on the effects of capital flows on the existence of the dot-com or the U.S. housing bubble, our paper has a more general focus of understanding the relationships between bubbles, capital flows and economic activities.

Finally, our paper also benefits from insights from the literature on global imbalances. The mechanism in our paper where the global asymmetry in financial development causes South-to-North capital flows is similar to that in Matsuyama (2005), Caballero et al. (2008), Mendoza et al. (2009), Song et al. (2011), Gourinchas and Jeanne (2013), Gourinchas and Rey (2013) and Buera and Shin (2015). For example, using a capital wedge analysis similar to the business cycle accounting method in Chari et al. (2007), Gourinchas and Jeanne (2013)

5 Also, see recent surveys by Barlevy (2012) and Miao (2014).
argue that the differences in domestic financial frictions, measured by wedges that distort saving and investment decisions, can help explain why capital flows from less developed to more developed countries. Chinn et al. (2014) provide some evidence for the prediction of these theories that economies with more developed financial markets have weaker current accounts. Through a panel analysis, they find that financial development (e.g., a stronger rule of law) is negatively related to the current account balance; emerging economies with less developed financial markets tend to have stronger current account surpluses, thus displaying a higher tendency for capital outflows; lagged real interest rates are negatively related to the current account balance. These findings are consistent with our theoretical prediction that economies with more developed financial markets and thus higher interest rates tend to be on the receiving end of capital flows.

The rest of the paper is organized as follows. To build intuition and establish autarky benchmarks, section 2 develops a closed economy model. Then, section 3 develops the full model in a world with two large open economies. Section 4 provides the main results. Section 5 provides concluding remarks.

2 Closed economy

It is instructive to begin with a closed economy model. We augment the classic SDT framework of rational bubbles with two features: heterogeneous productivity and a credit friction. Heterogeneous productivity give rise to natural borrowing and lending motives, while the credit friction allows for the possibility that bubbles crowd in investment and output. We first present the bubble-less benchmark and then introduce bubbles.

2.1 Bubble-less benchmark

Time is discrete and infinite, denoted by $t = 0, 1, 2, \ldots$. There are overlapping generations, each of which lives for two periods, “young age” and “old age” (except for the old generation in $t = 0$ who live for only one period). As in Bernanke and Gertler (1989), one can interpret the generational setting as representing the entry and exit of entrepreneurial agents and interpret each period as the length of a loan contract. Each generation consists of a continuum unit mass of infinitesimal agents. For simplicity, we assume that agents consume only in old age and are risk neutral. Young agents supply one unit of labor inelastically to firms and get wage income $W_t$. Old agents rent capital to firms at a rental rate $R^k_t$. For simplicity, we assume that capital depreciates completely after one period. These simplifying assumptions are relaxed in the robustness checks in the appendix. Firms are competitive and have a
Cobb-Douglas production function, \( Y_t = A_t K_t^\alpha L_t^{1-\alpha} \) with \( 0 < \alpha < 1 \), where \( Y_t \) is output, \( K_t \) is capital, \( L_t \) is labor and \( A_t = (1 + g)^t \) is the level of technological progress that grows at an exogenous gross growth rate \( 1 + g \geq 1 \). There is no friction in factor markets and thus factor prices are given by:

\[
W_t = (1 - \alpha) A_t K_t^\alpha, \quad R^k_t = \alpha A_t K_t^{\alpha-1}. \quad (2.1)
\]

As usual, it is convenient to detrend the exogenous growth component. For each equilibrium variable \( X_t \), we define the detrended variable \( x_t \) by \( x_t \equiv \frac{X_t}{A_t^{\frac{1}{(1-\alpha)}}} = \frac{X_t}{(1+g)^t} \). We abstract away from any uncertainty in the bubble-less benchmark for simplicity.

**Heterogeneous productivity:** Besides supplying labor, young agents also engage in entrepreneurial activities. A young agent can convert each unit of the consumption good into \( a \) units of capital in the subsequent period. The entrepreneurial productivity \( a \) is identically and independently distributed across agents according to a continuous distribution over a convex support \( \mathcal{A} \subseteq [0, \infty) \), whose cumulative distribution function \( F \) is strictly increasing and differentiable.

**Credit market and credit friction:** Young agents can borrow or lend each other and the loan is repaid when they grow old. Let \( R_{t+1} \) denote the interest rate on a loan between \( t \) and \( t + 1 \) and \( R^k_{t+1} \) denote the marginal product of capital in \( t + 1 \). In each period \( t \), given her net worth consisting of wage income \( W_t \), a young agent of type \( a \) chooses her net borrowing position \( D_t(a) \), where a negative position means that the agent is lending and a positive position means that the agent is borrowing, to produce capital stock \( K_{t+1}(a) \):

\[
K_{t+1}(a) = a [W_t + D_t(a)] . \quad (2.2)
\]

The consumption of the entrepreneur in period \( t + 1 \) is simply \( C_{t+1}(a) = R^k_{t+1} K_{t+1}(a) - R_{t+1} D_t(a) \).

Entrepreneurs face a leverage constraint:

\[
D_t(a) \leq \lambda_t(a) \frac{W_t}{\text{net worth}} , \quad (2.3)
\]

which states that each entrepreneur’s borrowing is limited by her net worth. The limit \( \lambda_t(a) \) places a constraint on the entrepreneur’s debt-over-net-worth (or leverage) ratio, where \( \lambda_t(a) \) is a weakly increasing function of \( a \). This formulation of credit market friction is defined as usual, note that even though \( K_{t+1} \) has time subscript \( t+1 \), it is determined in period \( t \). Similar for \( R_{t+1} \) and \( R^k_{t+1} \).
sufficiently general to envelope several types of credit constraints considered in the financial friction literature. For example, if one assumes \( \lambda_t(a) \equiv \frac{R^k_{t+1} \phi a}{R^k_{t+1} - R^k_{t+1} \phi a} \), where \( \phi \) is a positive constant, then the constraint \( (2.3) \) maps to a standard collateral constraint: \( R_{t+1} D_t(a) \leq \phi R^k_{t+1} K_{t+1}(a) \), which can arise when entrepreneurs can only pledge to repay in the next period at most a fraction \( \phi \) of the value of their asset (e.g., Kiyotaki and Moore 1997). Alternatively, if one assumes \( \lambda_t(a) \equiv \lambda \), where \( \lambda \geq 0 \) is a constant, then the constraint \( (2.3) \) maps to an analytically convenient form of collateral constraint, which states that the amount of credit is limited by the individual’s net worth and has been used extensively in the recent literature (e.g., Banerjee and Moll 2010, Buera and Shin 2013, Moll 2014). In general, a larger \( \lambda \) can be interpreted as representing an environment with less financial friction. We will often use this simple constant leverage constraint in examples throughout the paper.

In summary, a young agent of type \( a \) solves:

\[
\max_{\{K_{t+1}(a), D_t(a)\}} \frac{R^k_{t+1} K_{t+1}(a) - R_{t+1} D_t(a)}{}
\]

subject to capital production technology \( (2.2) \), the non-negativity constraint on capital \( K_{t+1}(a) \geq 0 \), and credit constraint \( (2.3) \).

**Equilibrium:** Given an initial aggregate capital stock \( K_0 \), a bubble-less equilibrium consists of a set of allocation \( \{D_t(a), K_{t+1}(a)\}_{a \in A} \) and prices \( \{R_{t+1}, R^k_{t+1}, W_t\} \) for each \( t \geq 0 \) such that given the prices, the set of allocation solves the problems of firms and young agents and the credit market clears in each \( t \geq 0 \):

\[
\int D_t(a) dF(a) = 0. \tag{2.4}
\]

We focus on stationary (or balanced growth path) equilibria, where the rates of return \( R_{t+1} \) and \( R^k_{t+1} \) and detrended variables \( d_t(a), k_{t+1}(a) \) and \( w_t \) are time-invariant.

**Solution**

With the heterogeneity in productivity, the optimal capital accumulation decision of agents with productivity \( a \) can be summarized by:

\[
\frac{K_{t+1}(a)}{W_t} \begin{cases} 
  = 0 & \text{if } R_{t+1} > aR^k_{t+1} \\
  \in [0, (1 + \lambda_t(a))a] & \text{if } R_{t+1} = aR^k_{t+1} \\
  = (1 + \lambda_t(a))a & \text{if } R_{t+1} < aR^k_{t+1}.
\end{cases} \tag{2.5}
\]
The first line states that if the interest rate is above the return from investing in capital, then agents lend only and does not accumulate capital. The second states that if they are equal, then agents are indifferent between lending and investing. Finally, the last states that if the interest rate is below the return from investing in capital, then agents invest in capital by borrowing up to the maximum amount that satisfies credit constraint (2.3).

Thus, in equilibrium, we have an endogenous segmentation of types into borrowers and lenders. In particular, there is a productivity threshold $\bar{a}_{nb,t} \in [0, 1]$ (the subscript stands for “no bubble”) such that:

\[
\frac{D_t(a)}{W_t} = \begin{cases} 
-1 & \text{if } a < \bar{a}_{nb,t} \\
\in [-1, \lambda_t(a)] & \text{if } a = \bar{a}_{nb,t} \\
\lambda_t(a) & \text{if } a > \bar{a}_{nb,t} 
\end{cases}
\]

(2.6)

In other words, agents with productivity strictly below $\bar{a}_{nb,t}$ choose not to invest in capital and become lenders, while those with productivity strictly above $\bar{a}_{nb,t}$ optimally borrow to the maximum subject to credit constraint (2.3) to invest in capital. Those with productivity $\bar{a}_{nb,t}$ are indifferent between investing in capital and lending and we call them marginal investors.

The indifference condition of the marginal investors leads to a no-arbitrage condition:

\[
R_{t+1} = \bar{a}_{nb,t} R_t^k.
\]

(2.7)

The left hand side is the return from lending and the right hand side is the return from investing in capital for marginal investors.

**Equilibrium dynamics:** From equations (2.5), (2.6) and (2.7), we can completely characterize the equilibrium dynamics. First, it is convenient to define the following functions for the investment rate $\mathcal{I}_t(\bar{a})$ and the capital accumulation rate $\mathcal{K}_t(\bar{a})$:

\[
\mathcal{I}_t(\bar{a}) \equiv \int_{a > \bar{a}} [1 + \lambda_t(a)] dF(a).
\]

\[
\mathcal{K}_t(\bar{a}) \equiv \int_{a > \bar{a}} [1 + \lambda_t(a)] a dF(a).
\]

Both of these functions are decreasing. Furthermore, let $\mathcal{R}_t(\bar{a})$ define the interest rate function:

\[
\mathcal{R}_t(\bar{a}) \equiv \frac{(1 + g)\alpha}{1 - \alpha} \frac{\bar{a}}{\mathcal{K}_t(\bar{a})},
\]

9
which is an increasing function and let $A$ be its inverse function, which provides threshold $\bar{a}$ as a function of the interest rate $R$:

$$
A_t(R) \equiv R_t^{-1}(R) = \bar{a}.
$$

This function is also increasing. Then, the investment rate function can be written as a function of the interest rate as $I_t(A_t(R))$. Because $I_t(\cdot)$ is decreasing and $A_t(\cdot)$ is increasing, the investment rate function is decreasing in the interest rate.

By aggregating the capital accumulation equation (2.5) and combining with the equilibrium wage from equation (2.1), we get the following law of motion for the detrended aggregate capital stock:

$$
\frac{k_{t+1}}{(1-\alpha)k_t^\alpha} = (1+g)K_t(\bar{a}_{nb,t}),
$$

(2.8)

where we have written $\lambda$ in place of $\lambda_t(a)$ for brevity.

Similarly, by aggregating the debt equation (2.6) and combining with the credit market clearing condition (2.4), we get the following identity that equates the aggregated investment (the left hand side) and the aggregated savings (the right hand side):

$$
\int_{a>\bar{a}_{nb,t}} [1 + \lambda_t(a)] dF(a) \times W_t = W_t.
$$

By canceling $W_t$ on both sides, we get an identity that equates the investment rate and the savings rate:

$$
I_t(\bar{a}_{nb,t}) = 1.
$$

(2.9)

This equation implicitly and uniquely determines the threshold $\bar{a}_t$.

The interest rate is determined by the combination of the no-arbitrage equation (2.7) and the marginal product of capital from equation (2.1):

$$
R_{t+1} = \bar{a}_{nb,t} \alpha k_{t+1}^{\alpha-1}.
$$

(2.10)

**Balanced growth path (BGP):** We can very conveniently derive the BGP of the economy from equations (2.8), (2.9) and (2.10) above. Then, from (2.8), we get the following

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\footnote{The savings rate is one in this model because we assume that agents only consume in old age. Of course the savings rate will be different under alternative specifications of the utility function. For example, of the utility function is $\ln c_y + \beta \ln c_o$, then the savings rate would be $\beta$. The uniqueness of $\bar{a}_t$ comes from the fact that the function $G(x)$ is decreasing and continuous in $x$ and that $\lim_{x \to \inf A} G(x) = \int [1 + \lambda(a)]dF(a) \geq \int dF(a) = 1$ and $\lim_{x \to \sup A} G(x) = 0$.}
expression for the detrended capital stock:

\[ k_{nb} = [(1 - \alpha)(1 + g)K(\bar{a}_{nb})]^{\frac{1}{1-\alpha}}. \]  

(2.11)

From (2.9) and (2.10), as well as the definition of the function \( \mathcal{A} \), the interest rate \( R_{nb} \) is implicitly and uniquely determined by the following savings-equal-to-investment equation:

\[ \mathcal{I}(\bar{a}_{nb}) = 1. \]  

(2.12)

and the productivity threshold is given by:

\[ \bar{a}_{nb} = \mathcal{A}(R_{nb}), \]  

(2.13)

The following lemma summarizes our analysis above:

**Lemma 1.** The bubble-less equilibrium features an endogenous segmentation of agents into borrowers and lenders. The equilibrium dynamics can be characterized by capital accumulation equation (2.5) and (2.8), debt equation (2.6), savings-equal-to-investment equation (2.9) and no-arbitrage equation (2.10).

The corresponding equations (2.11), (2.12) and (2.13) determine the capital stock, the cutoff productivity threshold and the interest rate on the BGP.

**Proof.** See the text above.

**Examples**

As the model is general, it is instructive to look at some special cases. Following Banerjee and Moll (2010), Buera and Shin (2013) and Moll (2014), we focus here on the most analytically tractable credit constraint, where the limit on the debt-over-net-worth ratio is a constant \( \lambda \geq 0 \).

In the first case, assume that the productivity distribution is the uniform distribution over \( [a_{\text{min}}, a_{\text{max}}] \subset [0, \infty) \). Then the productivity threshold, the interest rate and the detrended capital stock in the bubble-less BGP have closed-form expressions:

\[ \bar{a}_{nb} = \frac{a_{\text{min}} + \lambda a_{\text{max}}}{1 + \lambda} \]

\[ R_{nb} = \frac{2(1 + g)\alpha}{1 - \alpha} \frac{a_{\text{min}} + \lambda a_{\text{max}}}{a_{\text{min}} + (1 + 2\lambda)a_{\text{max}}} \]

\[ k_{nb} = (1 - \alpha)(1 + g)\frac{a_{\text{min}} + (1 + 2\lambda)a_{\text{max}}}{1 + (1 + 2\lambda)} \]
It can be seen that $\bar{a}_{nb}$, $R_{nb}$ and $k_{nb}$ are increasing in the leverage limit $\lambda$, as a larger $\lambda$ allows more resources to be shifted towards more productive agents (reflected by a higher weight on $a_{\text{max}}$ in the expressions above). Furthermore, a decrease in the distribution $F$ in the first order stochastic dominance sense (in this case, this means a decrease in $a_{\text{max}}$ and/or $a_{\text{min}}$) is also associated with a decrease in $\bar{a}_{nb}$, $R_{nb}$ and $k_{nb}$.

In the second case, assume that the productivity distribution is the log normal distribution. The equilibrium can only be solved numerically. Figure 1 plots the two key functions $A(R)$ and $I(A(R))$ that determine the BGP. It can be seen that, as stated earlier, the threshold function $A$ are both increasing, while the investment function $I$ is decreasing. The bottom panel shows how the interest rate $R_{nb}$ is determined by the intersection of the downward sloping curve representing the investment rate function $I(A(R))$ and the flat line representing the savings rate of one. The panel also illustrates how the investment curve shifts downward (see the dashed line), leading to a decrease in the equilibrium interest, when there is either a decrease in $\lambda$ or a decrease in $F$ in the first order dominance sense. We will revisit this comparative statics in the open economy section.

Fig. 1: Plots of threshold function $A(R)$ and investment rate function $I(A(R))$ (log-normal productivity distribution).
2.2 Bubbles

We now introduce asset bubbles. Following the SDT framework, we model a (pure) bubble as an asset that pays no dividend and thus has a zero fundamental value, but is traded at a positive price. The only reason an individual purchases a bubble is that he or she expects to be able to resell it later. As in Blanchard and Watson (1982), a bubble can be interpreted as attached to the value of a firm or real estate.\footnote{For instance, to model bubbles attached to values of firms or firms’ equity, define the value of a competitive firm \( i \) as:
\[
B_{i,t} = \max_{k_{i,t}, l_{i,t}} \{ k_{i,t} \alpha l_{i,t}^{1-\alpha} - w_{i,t} l_{i,t} - R_{i,t} k_{i,t} + \frac{B_{i,t+1}^{k} + 1}{R_{t+1}} \},
\]
The last term \( B_{i,t+1}/R_{t+1} \) is the value of the firm in period \( t+1 \) discounted by the world interest rate \( R_{t+1} \). Because the profit of a competitive firm is zero, the fundamental value of a firm is zero. Thus
\[
B_{i,t} = \frac{B_{i,t+1}}{R_{t+1}},
\]
which implies that if \( B_{i,t} > 0 \), then there is a bubble in the value of firm \( i \) and the bubble is akin to a pyramid scheme. To model bubbles attached to real estate, assume that the price of a house is \( B_{t} = v + \frac{B_{t+1}}{R_{t+1}} \), where \( v \geq 0 \) is the utility dividend from owning a house. As \( v \to 0 \), the housing price approaches a pure bubble.\footnote{There could be equilibria in which the relative size of bubbles is time-varying and denoted as \( n_{t} \). Our analysis can be easily applied to such cases.}}

To model the expansionary effect of bubbles on the aggregate debt and investment, we follow Martin and Ventura (2012) and introduce bubble creation. Each young agent can create one unit of new bubble assets and for simplicity we assume that bubble creation is costless (bubble creation is thus effectively a wealth shock). Let \( B_{t}^{N} \) denote the value of the portfolio that contains all old bubbles and let \( B_{t}^{N} \) denote the portfolio that contains all new bubbles created in period \( t \). By definition, the total value of all bubbles \( B_{t} \) is \( B_{t} = B_{t}^{O} + B_{t}^{N} \).

For simplicity, as in Martin and Ventura (2012), throughout the paper, we focus on bubble equilibria in which the relative size of new bubbles \( n \) is constant:\footnote{Our analysis can be easily applied to such cases.}

\[
B_{t}^{N} = n B_{t}, \quad 0 \leq n < 1.
\]

Consequently, the value of old bubbles is \( B_{t}^{O} = (1 - n) B_{t} \). One interpretation of bubble creation is that young agents are entrepreneurs who create new firms or structures and there are bubbles in the value of these new firms. The creation of new tech firms during the dot-com boom is an example of bubble creation. The construction of new houses during the housing bubble episode in the 2000s, is another example of bubble creation.

With bubble creation, the net worth of a young agent in each period is \( W_{t} + n B_{t} \), instead of \( W_{t} \) as in the bubble-less benchmark. The constraint (2.3) on the debt-over-net-worth ratio
is thus replaced by:

$$D_t(a) \leq \lambda_t(a) \left( W_t + nB_t \right).$$  \hspace{1cm} (2.14)

For convenience, we denote the aggregate bubble-over-net-worth ratio (or bubble ratio for brevity) by:

$$\beta_t \equiv \frac{B_t}{W_t + nB_t}.$$  

Bubbles are fragile and they require the coordination of expectation across generations: one would buy bubbles only if they expect someone else would buy them in the future. To model this fragility, we follow Weil (1987) and assume that in any period, the value of all existing bubbles exogenously can crash to zero with an exogenous probability $p \in [0,1)$. Agents rationally discount the risk that the bubbles crash. Once collapsed, bubbles are not expected to take place again in the future.

The optimization problem of a young agent of entrepreneurial productivity $a$ is to choose a portfolio consisting of investment in capital $I_t(a)$, net debt position $D_t(a)$ and value of bubble holding $B_t(a)$, to maximize the expected consumption in the subsequent period:

$$\max_{\{I_t(a), D_t(a), B_t(a)\}} R_{t+1}^k + (1-p) \frac{B_{t+1}^O}{B_t} B_t(a) - R_{t+1} D_t(a),$$

subject to the capital production technology:

$$K_{t+1}(a) = aI_t(a),$$

the budget constraint:

$$I_t(a) + B_t(a) = W_t + nB_t + D_t(a),$$

the non-negativity constraints on capital and bubbles, $K_{t+1}(a) \geq 0$ and $B_t(a) \geq 0$ and credit constraint (2.14). In the maximand, $R_{t+1}^k K_{t+1}(a)$ is the return from capital, $(1-p) \frac{B_{t+1}^O}{B_t}$ is the expected return from purchasing one unit of bubble in period $t$ at price $B_t$ and reselling at price $B_{t+1}^O$ in the next period if bubbles do not crash and $R_{t+1} D_t(a)$ is the repayment of debt. In the budget constraint, $I_t(a)$ is the expense required to produce $K_{t+1}(a)$ units of capital, $B_t$ is the expenditure on purchasing bubbles, $W_t + nB_t$ is the agent’s net worth, consisting of wage income and the value of newly created bubbles and $D_t(a)$ is the net borrowing.

**Equilibrium:** Given an initial aggregate capital stock $K_0 > 0$, a (stochastic) bubble equilibrium consists of allocations $\{B_t(a), D_t(a), K_{t+1}(a)\}_{a \in A}$, prices $R_t$, $R_{t+1}^k$ and $W_t$, and value of bubbles $B_t > 0$ for each period $t \geq 0$, such that: (i) given prices, the allocations solve the optimization problems of firms and agents, (ii) the credit market clearing condition (2.14)
holds in each period and (iii) the bubble market clears in each period:

\[ \int B_t(a) dF(a) = B_t. \] (2.16)

A bubble balanced growth path is a stochastic bubble equilibrium in which the rates of return, \( R_t \) and \( R^k_t \) and detrended variables, \( b_t, d_t(a), k_{t+1}(a) \) and \( w_t \), are time-invariant.

**Solution**

As in the bubble-less benchmark, a bubble equilibrium is characterized by an endogenous segmentation of types into borrowers and lenders at a productivity threshold \( \bar{a}_{b,t} \in A \). Similar to equations (2.5) and (2.6), the capital-over-net-worth ratio across agents can be summarized by:

\[
\frac{K_{t+1}(a)}{W_t + nB_t} = \begin{cases} 
0 & \text{if } a < \bar{a}_{b,t} \\
(1 + \lambda_t(a))a & \text{if } a = \bar{a}_{b,t} \\
(1 + \lambda_t(a))a & \text{if } a > \bar{a}_{b,t}
\end{cases}
\] (2.17)

and the leverage ratio across agents can be summarized by:

\[
\frac{D_t(a)}{W_t + nB_t} = \begin{cases} 
0 & \text{if } a < \bar{a}_{b,t} \\
[0, \lambda_t(a)] & \text{if } a = \bar{a}_{b,t} \\
\lambda_t(a) & \text{if } a > \bar{a}_{b,t}
\end{cases}
\] (2.18)

Equation (2.17) shows how bubble creation increases young agents’ net worth from \( W_t \) to \( W_t + nB_t \), as these agents can sell their newly created bubbles at market value \( nB_t \). As a consequence, bubbles have a crowd-in effect on capital investment, by raising the net worth of young agents and consequently their ability to borrow from the credit market.

The agents with type \( a = \bar{a}_{b,t} \) are marginal investors, as they are indifferent between lending, investing in capital, or investing in bubbles. Their indifference yields no-arbitrage conditions:

\[
R_{t+1} = \bar{a}_{b,t} R^k_{t+1} = \frac{(1 - p)(1 - n)(1 + g)b_{t+1}}{b_t}. \] (2.19)

The first term is the interest rate, the second term is the return from capital for marginal investors and the last term is the expected return from bubble speculation. For agents with \( a > \bar{a}_{b,t} \), the return from capital \( aR^k_{t+1} \) is greater than the return from lending and the return from bubble speculation. Thus, only those with \( a \leq \bar{a}_{b,t} \) purchase bubbles.

*Equilibrium dynamics:* From equations (2.17), (2.18) and (2.19), the equilibrium dy-
Dynamics can be characterized as follows. By aggregating the individual capital accumulation equation (2.17) and combining with the equilibrium wage from equation (2.1), we get the following law of motion for the detrended aggregate capital stock:

\[
\frac{k_{t+1}}{(1 - \alpha)k_t^\alpha + nb_t} = (1 + g)K_t(\bar{a}_{b,t}).
\]

Note that the difference between this equation and the counterpart equation (2.20) in the bubble-less benchmark lies in the denominator on the left hand side: the (detrended) net worth of a young agent in period \( t \) is no longer \((1 - \alpha)k_t^\alpha\) but \((1 - \alpha)k_t^\alpha + nb_t\), due to bubble creation. It is more convenient to rewrite this equation by using the bubble ratio \( \beta_t \):

\[
\frac{(1 - n\beta_t)k_{t+1}}{(1 - \alpha)k_t^\alpha} = (1 + g)K_t(\bar{a}_{b,t}).
\] (2.20)

By aggregating the debt equation (2.18) and combining with the credit market clearing condition, we get an identity that equates the aggregated investment in capital and bubble (the left hand side) and the aggregated savings (the right hand side):

\[
\underbrace{I_t(\bar{a}_{b,t})(W_t + nB_t)}_{\text{capital investment}} + \underbrace{B_t}_{\text{bubble investment}} = \underbrace{W_t + nB_t}_{\text{savings}}.
\]

By dividing the net worth \( W_t + nB_t \) on both sides, we get an identity between the investment rate and the savings rate:

\[
I_t(\bar{a}_{b,t}) + \beta_t = 1.
\] (2.21)

This equation differs from the counterpart equation (2.9) in the bubble-less benchmark in an important aspect: the presence of the bubble-over-net-worth term \( \beta_t \), which has to be positive in the bubble equilibrium. This captures the fact that in a bubble equilibrium, total savings can be channeled into either capital investment or bubble investment. Thus, a direct comparison between the two equations imply that \( I_t(\bar{a}_{b,t}) = 1 - \beta_t < 1 = I_t(\bar{a}_{nb,t}) \).

Recall that the capital investment rate function \( I_t(\bar{a}) \equiv \int_{a > \bar{a}} [1 + \lambda_t(a)] dF(a) \) is decreasing. Therefore, an interesting implication immediately follows from equations (2.9) and (2.21): the threshold in the bubble equilibrium must be larger than the threshold in the bubble-less equilibrium, i.e.,:

\[
\bar{a}_{b,t} > \bar{a}_{nb,t}.
\] (2.22)

Intuitively, as bubbles provide a new investment opportunity, some agents find it optimal to stop producing capital and instead switch to bubble speculation, causing the productivity threshold to rise from \( \bar{a}_{nb,t} \) to \( \bar{a}_{b,t} \).
From the no-arbitrage (2.19) and the marginal product of capital from equation (2.1), we have the following expression for the interest rate:

$$R_{t+1} = \bar{a}_{b,t} \alpha k_{t+1}^{\alpha - 1},$$

which is identical to equation (2.10) in the bubble-less benchmark.

Finally, the evolution of the detrended bubble value follows immediately from equation (2.19):

$$\frac{(1 + g)b_{t+1}}{b_t} = \frac{R_{t+1}}{(1 - p)(1 - n)}.$$  

(2.24)

**Balanced growth path:** From equations (2.20) to (2.24) above, we can characterize the BGP. From (2.20), the detrended capital stock satisfies:

$$(1 - n\beta)k_b = \left[(1 - \alpha)(1 + g)K(\bar{a}_b)\right]^{\frac{1}{1 - \alpha}}.$$  

(2.25)

From the no-arbitrage equation (2.19), it immediately follows that the interest rate in the BGP is simply proportional to the growth rate:

$$R_b = (1 - p)(1 - n)(1 + g).$$  

(2.26)

From the interest rate equation (2.23), the investment-savings equation (2.9) can be rewritten as:

$$1 - I(A(\bar{a}_b)) = \beta,$$

(2.27)

which implicitly determines the bubble ratio $\beta$. By combining (2.23) with capital equation (2.25) and the definition of the function $A$, the productivity threshold is simply:

$$\bar{a}_b = A\left(\frac{R_b}{1 - n\beta}\right).$$

(2.28)

We summarize our characterization of the bubble equilibrium by the following lemma:

**Lemma 2.** The bubble equilibrium is characterized by an endogenous segmentation of agents into borrowers and lenders at a threshold $\bar{a}_{b,t} > \bar{a}_{nb,t}$.

The equilibrium dynamics can be characterized by the capital accumulation equation (2.20), savings-equal-to-investment equation (2.21), interest rate equation (2.23) and bubble growth equation (2.24).

The corresponding set of equations (2.25), (2.26), (2.27) and (2.28) determine the bubble BGP.
Proof. See the text above.

Fig. 2: Excess savings function $SI(R) \equiv 1 - I(A(R))$ and determination of bubble-over-networth ratio $\beta$.

We can use graphical analysis to gain more intuition behind the determination of the bubble BGP. First, let us consider the case without bubble creation, i.e., $n = 0$. The left panel of figure 2 illustrates the savings-investment equation (2.27). (Both panels of the figure are plotted numerically under the assumption of a log-normal productivity distribution.) The upward sloping solid line plots the function $SI(R) \equiv 1 - I(A(R))$, which is equal to the savings rate minus the capital investment rate at each interest rate $R$. At any interest rate $R$, the savings rate is inelastic at 1, while the capital investment rate is $I(A(R))$, as only agents whose productivity is above $A(R)$ would invest in accumulating capital. The curve intersects the horizontal axis at $R = R_{nb}$, reflecting equation (2.12), which states that in the bubble-less economy, the credit market must clear at the equilibrium interest rate $R_{nb}$.

However, in the bubble economy, due to the no-arbitrage equation between lending and investing in the risky bubble market, the interest rate is pinned down at $R = R_b$ (equation (2.26)). At this interest rate, the capital investment rate is only $I(A(R_b))$, leading to an “excess savings rate” of $SI(R_b)$. In equilibrium, this excess savings must be absorbed by the investment in the bubble, i.e., $\beta = 1 - I(A(R_b))$.

The left panel of the figure also highlights an important result: when $n = 0$, the bubble ratio is positive if and only if $R_{nb} < R_b$. This corresponds to a standard result in the rational bubble literature that bubbles can only exist in a low interest rate environment, i.e., the interest rate in the bubble-less equilibrium is sufficiently low.
Now, let us consider the more general case of \( n \geq 0 \). The savings-investment equation \((2.27)\) gives the bubble ratio \( \beta \) solution to \( x = 1 - \mathcal{I}(\mathcal{A}(\frac{R_b}{1-nx})) \), or equivalently \( x = SI(\frac{R_b}{1-nx}) \). The right panel of figure 2 illustrates the solution to this equation as the intersection of the dashing 45 degree line, which represents the left hand side, and the upward sloping solid line, which represents the right hand side. The curve is upward sloping because when \( n > 0 \), the function \( SI(\frac{R_b}{1-nx}) \) is increasing in \( x \). (Note that when \( n = 0 \), the curve representing \( SI(\frac{R_b}{1-nx}) \) is simply a straight horizontal line, as illustrated in the figure.)

The right panel also helps us understand the existence condition of the bubble when \( n \geq 0 \). When \( n = 0 \), the excess savings curve \( SI(\frac{R_b}{1-nx}) \) is flat. When \( n > 0 \), the curve is upward sloping and the steepness of the slope increases with \( n \). For any given productivity distribution \( F \), we assume that \( n \) is not too large relative to the slope of the CDF function \( F \), so that the curve \( SI(\frac{R_b}{1-nx}) \) is not as steep as the 45 degree line. Formally, we assume the following elasticity condition: \(^{10}\)

\[
\frac{d}{dx} SI \left( \frac{R_b}{1-nx} \right) < 1, \tag{2.30}
\]

for all \( 0 < x < 1 \). Then for the \( SI(\frac{R_b}{1-nx}) \) curve and the 45 degree line to intersect at some \( x = \beta \in (0, 1) \), it can be shown that a necessary and sufficient condition is \( SI(\frac{R_b}{1-nx})\big|_{x=0} > 0 \), or equivalently, \( SI(R_b) > SI(R_{nb}) \). Since \( SI \) is increasing, this is in turn is equivalent to \( R_b > R_{nb} \). In other words, as long as the elasticity condition \((2.30)\) is satisfied, the existence condition for the bubble is the standard low interest rate condition.

The discussions above is formalized by the following result:

**Proposition 1.** [Existence] Assume the elasticity condition \((2.30)\) holds for all \( 0 < x < 1 \). Then there exists a bubble BGP if and only if the interest rate on the bubble-less BGP is low:

\[
R_{nb} < \left( 1 - p \right) \left( 1 - n \right) \left( 1 + g \right) \frac{1}{R_b}. \tag{2.31}
\]

**Proof.** Appendix. \( \square \)

**Remark 1.** Assumption \((2.30)\) obviously holds when \( n = 0 \) and more generally when \( n \) is not too large. We numerically verify that for most standard continuous productivity distributions, such as the uniform distribution or the log-normal distribution, the set of

\[
(1 + \lambda)(1 - p)(1 - n) F'(\mathcal{A}(\frac{R_b}{1-nx})) A'(\frac{R_b}{1-nx}) \frac{n}{(1-nx)^2} < 1, \tag{2.29}
\]

for all \( 0 < x < 1 \).
parameters such that (2.30) holds and such that expansionary bubble BGP exists (i.e., \( k_b > k_{nb} \), which requires \( n \) to be not too small) is non-empty.

3 Open economies

We now extend the closed economy model to an environment with two large open economies. We first describe the model, solve the bubble-less benchmark, then analyze how financial integration affects on the existence of bubbles and finally show how bubbles affect capital flows.

Consider two open economies, called the North and the South, each having the same structure as described in the closed model. We denote variables in the South with a star (\( \ast \)) and denote corresponding variables in the closed economy with a superscript \( c \) (for example, \( R^c_t \) and \( R^c_{\ast t} \) represent the interest rate in the closed North and closed South, respectively). The two economies have the same TFP growth (\( g = g^\ast \)) and the same Cobb-Douglas production function (\( \alpha = \alpha^\ast \)). They can differ in the leverage limits \( \lambda_t(\cdot) \) and \( \lambda^\ast_t(\cdot) \), which capture the extent of financial market imperfections and in the productivity distributions \( F \) and \( F^\ast \).

For simplicity, we assume that agents and firms can trade capital, labor and bubbles in the domestic markets only. However, the credit market is perfectly integrated: agents can freely borrow and lend across borders. Hence, the following interest rate parity must hold:

\[
R_t + 1 = R^\ast_{t + 1},
\]

and the world credit market must clear:

\[
\int D_t(a) dF(a) + \int D^\ast_t(a) dF^\ast(a) = 0.
\]

3.1 Bubble-less benchmark

We first study the bubble-less benchmark. Given initial aggregate capital stocks \( K_0, K^\ast_0 \), a bubble-less equilibrium in the open economies consists of allocations \( \{D_t(a), K_{t+1}(a), D^\ast_t(a), K^\ast_{t+1}(a)\}_{a \in \mathbb{A}} \) and prices \( \{R_t, R^\ast_t, R^k_t, R^k_{\ast t}, W_t, W^\ast_t\} \) for each period \( t \geq 0 \) such that: (i) given prices, the allocations solve firms’ and agents’ optimization problems in each country, (ii) interest rate parity condition (3.1) holds and (iii) integrated credit market clearing condition (3.2) holds.

Equilibrium dynamics: As the dynamics is symmetric between the two economies, it is sufficient to focus on the North. The equilibrium dynamics is similar to that in section 2. The economy is segmented into borrowers and lenders at an endogenous productivity
threshold \( \bar{a}_{nb,t} \). In each period \( t \), given the capital stock \( K_t \) as a state variable, the law of motion of capital is given by (2.8) and the no-arbitrage condition that equates the interest rate and the return from capital investment is (2.10).

However, an important difference lies in the credit market clearing condition. In the closed economy, the condition is simply \( I_t(\bar{a}_{nb,t}) = 1 \), where the left hand side is the investment rate and the right hand side is the savings rate (equation (2.9)). In the open economy, the corresponding condition is:

\[
I_t(\bar{a}_{nb,t}) + \mu_t^* I_t^*(\bar{a}_{nb,t}^*) = 1 + \mu_t^*,
\]

where the left hand side is the weighted sum of the investment rates across both economies and the right hand side is the weighted sum of the savings rates. The rates in the South is weighted by the size of the South’s aggregated net worth (or equivalently, GDP) relative to that of the North\(^{11}\)

\[
\mu_t^* \equiv \frac{w_t^*}{w_t} = \left( \frac{k_t^*}{k_t} \right)^\alpha.
\]

By using the no-arbitrage equation for both economies \( \bar{a}_{nb,t} R_{t+1}^k = R_{t+1} = \bar{a}_{nb,t}^* R_{t+1}^{k^*} \) and the factor price equation for the rental rate of capital \( R_{t+1}^k \) and \( R_{t+1}^{k^*} \), it is straightforward to show that the weight can be rewritten as:

\[
\mu_t^* = \left( \frac{\bar{a}_{nb,t-1}^*}{\bar{a}_{nb,t-1}} \right)^{\frac{\alpha}{1-\alpha}}.
\]

It is convenient to rewrite the credit market clearing equation above as:

\[
(1 - I_t(\bar{a}_{nb,t})) + \mu_t^* (1 - I_t^*(\bar{a}_{nb,t}^*)) = 0, \tag{3.3}
\]

where the left hand side is the weighted sum of the excess savings rates (defined as the savings rate minus the investment rate). The equation states that the world interest rate \( R_{nb,t} \) must clear the world credit market.

**Balanced growth path:** From the equilibrium dynamics, the characterization of the BGP

\[\int_{a > \bar{a}_{nb,t}} (1 + \lambda_t(a)) dF(a) \times W_t + \int_{a > \bar{a}_{nb,t}^*} (1 + \lambda_t^*(a)) dF^*(a) \times W_t^* = W_t + W_t^*, \]

and the factor price equations: \( W_t = (1 - \alpha) A_t K_t^\alpha \) and \( W_t^* = (1 - \alpha) A_t K_t^{*\alpha} \).
immediately follows. The detrended aggregate capital stocks are given by:

\[ k_{nb} = [(1 - \alpha)(1 + g)K(\bar{a}_{nb})]^{\frac{1}{1-\alpha}}, \]
\[ k^*_{nb} = [(1 - \alpha)(1 + g)K^*(\bar{a}^*_{nb})]^{\frac{1}{1-\alpha}}, \]

as in equation (2.11) from the closed economy model. The productivity threshold is given by:

\[ \bar{a}_{nb} = A(R_{nb}), \]
\[ \bar{a}^*_{nb} = A^*(R_{nb}), \]

as in (2.13). From (3.3), the world interest rate \( R_{nb} \) solves the world credit market clearing condition:

\[ \left( 1 - I(A(R_{nb})) \right) + \left( \frac{A^*(R_{nb})}{A(R_{nb})} \right)^{\frac{\alpha}{1-\alpha}} \left( 1 - I^*(A^*(R_{nb})) \right) = 0. \]

Note that since the excess savings functions \( SI \) and \( SI^* \) are increasing and since \( SI(R_{nb}^c) = SI^*(R_{nb}^c) = 0 \) (recall the credit market clearing conditions in closed economies), it immediately follows from (3.6) that the world interest rate \( R_{nb} \) must lie between the closed economy interest rates \( R_{nb}^c \) and \( R_{nb}^c \).

The following lemma summarizes our analysis:

**Lemma 3.** The bubble-less equilibrium dynamics of aggregate variables in the North can be characterized by capital accumulation equation and (2.8), no-arbitrage equation (2.10) and world credit market clearing equation (3.3). The dynamics is symmetric in the South.

The corresponding equations (3.4), (3.5) and (3.6) determine the detrended aggregate capital stock in the North, the cutoff productivity threshold in the North, and the world interest rate \( R_{nb} \) on the BGP, respectively.

The world interest rate \( R_{nb} \) lies between the closed economy interest rates \( R_{nb} \), i.e.,

\[ \min \{ R_{nb}^c, R_{nb}^c \} \leq R_{nb} \leq \max \{ R_{nb}^c, R_{nb}^c \}, \]

and the inequalities are strict if \( R_{nb}^c \neq R_{nb}^c \).

**Proof.** See text above.

Figure 3 is a Metzler diagram that illustrates the determination of the world interest rate \( R_{nb} \) in the bubble-less BGP and compare it against the interest rates in the closed economy. To generate the figure, we focus on the simplest case: the productivity distributions in the

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\(^{12}\) To see this, suppose on the contrary that \( R_{nb} < \min \{ R_{nb}^c, R_{nb}^c \} \). Then both \( SI(R_{nb}) \) and \( SI^*(R_{nb}) \) are negative. This contradicts (3.6). Similarly, one would get a contradiction if \( R_{nb} > \max \{ R_{nb}^c, R_{nb}^c \} \). Thus \( \min \{ R_{nb}^c, R_{nb}^c \} \leq R_{nb} \leq \max \{ R_{nb}^c, R_{nb}^c \} \). Note that if \( R_{nb}^c \neq R_{nb}^c \), then the inequalities are strict.
two economies are the same log-normal distribution; the leverage constraint (2.3) takes the convenient form of two positive constants $\lambda$ and $\lambda^*$, where we set $\lambda > \lambda^*$ to reflect an assumption that the North is more financially developed than the South.

The top panel plots the savings rate curve, which is flat at one for both economies, and the investment rates curve $I(A(R))$ and $I^*(A((R)))$, which are the two downward sloping curves. As we discussed in the closed economy section 2, the intersection of the investment curve in each economy and the savings curve determines the interest rates $R_{cb}^c$ and $R_{cb}^{c*}$ for the closed economies of the North and of the South, respectively. As we assume the North is more financially developed, it can be seen that the autarky interest rate is higher in the North, i.e., $R_{cb}^c > R_{cb}^{c*}$.

How is the world interest rate $R_{nb}$ determined? The bottom panel plots the weighted sum $SI_{world} \equiv SI(R) + \left(\frac{A^*(R)}{A(R)}\right)^{1-\alpha} SI^*(R)$ of the excess savings rates. From world credit clearing equation (3.6), we know that $R_{nb}$ is the intersection of this curve and the horizontal axis. As seen in the figure, the world interest rate $R_{nb}$ lies between the autarky interest rates.
Furthermore, we can see that at the world interest rate $R_{nb}$, the North runs a trade deficit, while the South runs a trade deficit. This is because the excess savings rate is negative for the North:

$$SI(R_{nb}) < SI(R_{nb}^c) = 0$$

and positive for the South:

$$SI^*(R_{nb}) > SI^*(R_{nb}^c) = 0.$$
according to:
\[
\frac{(1 - n\beta_t)k_{t+1}}{(1 - \alpha)k_t^*} = (1 + g)K_t(a_{b,t}),
\]  
(3.7)
as in equation (2.20) from the closed economy model. As there is no bubble (and no bubble creation) in the South, the capital stock there evolves according to:
\[
\frac{k^{*}_{t+1}}{(1 - \alpha)k_t^{*\alpha}} = (1 + g)K_t^*(\bar{a}_{b,t}^*),
\]  
(3.8)
as in equation (2.8) from the closed bubble-less model. The indifference condition for the marginal investors in the North yields a no-arbitrage equation:
\[
R_{t+1} = \bar{a}_{b,t}R_{k_{t+1}} = \frac{(1 - p)(1 - n)(1 + g)b_{t+1}}{b_t},
\]  
as in equation (2.19). The corresponding equation in the South is:
\[
R_{t+1} = \bar{a}_{b,t}^*R_{k_{t+1}}.
\]  
Together, they imply a single no-arbitrage condition:
\[
R_{t+1} = \bar{a}_{b,t}R_{k_{t+1}} = \bar{a}_{b,t}^*R_{k_{t+1}} = \frac{(1 - p)(1 - n)(1 + g)b_{t+1}}{b_t},
\]  
(3.9)
However, unlike in the closed economy, the world credit market clearing condition is:
\[
\mathcal{I}_t(\bar{a}_{b,t}) + \beta_t + \mu_{b,t}^*\mathcal{I}_t^*(\bar{a}_{b,t}^*) = 1 + \mu_{b,t}^*,
\]  
(3.10)
where the left hand side is the weighted sum of the investment rates and the bubble ratio \(\beta\), while the right hand side is the weighted sum of the savings rates. The rates in the South is weighted by the relative size of its net worth:
\[
\mu_{b,t}^* \equiv \frac{w_t^*}{w_t + nb_t}.
\]  
By the no-arbitrage equations for both economies, it is straightforward to show that the weight can be rewritten as:
\[
\mu_{b,t}^* = \left(\frac{\bar{a}_{b,t-1}}{\bar{a}_{b,t-1}}\right)^{\frac{\alpha}{\alpha}}(1 - n\beta_t).
\]
Credit clearing equation (3.10) can be rewritten as:

\[(1 - I_t(\bar{a}_{b,t})) + \mu_{t,t}^*(1 - I_{b,t}^*(\bar{a}_{b,t})) = \beta_t, \quad (3.11)\]

which is the open economy counterpart to equation (2.27) in the closed economy section.

Balanced growth path: From the equilibrium dynamics above, the bubble BGP of the open economy can be summarized as follows. The detrended aggregate capital stocks are given by:

\[
(1 - n\beta)k_b = [(1 - \alpha)(1 + g)\mathcal{K}(\bar{a}_b)]^{\frac{1}{1-\alpha}} \quad (3.12)
\]

\[
k_b^* = [(1 - \alpha)(1 + g)\mathcal{K}^*(\bar{a}_b^*)]^{\frac{1}{1-\alpha}}.
\]

From the no-arbitrage equation, the world interest rate is simply:

\[R_b = (1 - p)(1 - n)(1 + g),\]

as in (2.26), where the right hand side is the expected return rate from bubble speculation. The productivity thresholds are given by:

\[
\bar{a}_b = A\left(\frac{R_b}{1 - n\beta}\right), \quad (3.13)
\]

\[
\bar{a}_b^* = A^*(R_b).
\]

The bubble ratio is determined by the world credit market clearing condition (3.11) as

\[
\left(1 - I(\bar{a}_b)\right) + \left(\frac{A^*(R_b)}{\mathcal{A}\left(\frac{R_b}{1 - n\beta}\right)}\right)^{\frac{1}{1-\alpha}}(1 - n\beta) \cdot (1 - I^*(\bar{a}_b^*)) = \beta. \quad (3.14)
\]

In summary, we have established:

**Lemma 4.** Assuming a bubble in the North. The equilibrium dynamics of aggregate variables can be characterized by capital accumulation equation (3.7) for the North and (3.8) for the South, no-arbitrage equation (3.9) and world credit market clearing equation (3.11).

On the BGP, the corresponding equations (3.12), (3.13) and (3.14) determine the detrended aggregate capital stock, the cutoff productivity thresholds and the bubble ratio, respectively.

**Proof.** See text above.
4 Bubbles and capital flows

Having established a general framework of bubbles in both closed and open economies, we are now ready to establish the main results of the paper. We will show that under general conditions, there is a reinforcing relationship between bubbles and capital flows. Throughout this section we focus on the situation where the North is more financially developed than the South (represented by a higher autarky interest rate in the North) and focus on the possibility of bubbles in the North. We have in mind the North representing the U.S. economy, the South representing China, and financial integration representing the integration of the Chinese economy into the global financial market.

4.1 Effects of capital flows on bubble

We begin with analyzing the effects of capital flows on bubbles. To get intuition, let us come back to the special case with the log-normal productivity distribution and constant leverage constraints \( \lambda > \lambda^* \) in section 3.1.

First, assume \( n = 0 \). The left panel of figure 4 plots the weighted sum \( SI_{\text{world}}(R) \equiv SI(R) + \left( \frac{A^*(R)}{A(R)} \right)^{1-\alpha} SI^*(R) \) of the excess savings rate functions (the red solid line). This is the same curve as plotted in the bottom panel of figure 3. Furthermore, the panel also plots the excess savings rate functions \( SI(R) \) and \( SI^*(R) \) in each economy. All the three curves are upward sloping, as the functions are increasing in \( R \). The three curves intersect the horizontal axis at the respective interest rates \( R_{nb} \) (interest rate under financial integration), \( R_{nb}^c \) (interest rate in the closed North) and \( R_{nb}^s \) (interest rate in the closed South). Because the North is more financially developed, we know from section 3.1 that

\[ R_{nb}^s < R_{nb} < R_{nb}^c. \]

The interest rate in the BGP is \( R_b = (1-p)(1-n)g \).

Then as in the closed economy section, a bubble can exist under financial integration if and only if \( R_b > R_{nb} \). To see this, when \( R_b > R_{nb} \), the total excess savings (illustrated by the red solid line) is positive: \( SI_{\text{world}}(R_b) > SI_{\text{world}}(R_{nb}) = 0 \). That is, at the interest rate \( R_b \), the total investment in capital stock cannot fully absorb the total savings in the world economy. The excess savings must be absorbed by speculative investment in the bubble asset. That is, when \( n = 0 \), the bubble ratio \( \beta \) is simply equal to \( SI_{\text{world}}(R_b) \).

How does financial integration affect the bubble in the North? There are two effects. The first effect is on the existence condition for bubbles in the North. Recall that when \( n = 0 \), a bubble BGP exists in the closed North if and only if the interest rate in the closed North is
low: \( R_{nb}^c < R_b \). Similarly, as our analysis above suggests, the bubble exists in the open North if and only if the *world* interest rate is low: \( R_{nb} < R_b \). However, as the integration of the North with the less developed South leads to a decrease in the interest rate, i.e., \( R_{nb} < R_{nb}^c \), it then immediately follows that financial integration *relaxes* the existence condition for bubbles in the North. In other words, capital inflows from the rest of the world facilitates the existence of bubbles in the North.

The second effect is on the *size* of the bubble (conditional on bubble existence). From the figure, it is apparent that the bubble ratio in the closed economy is smaller: \( \beta^c < \beta \). Formally, this inequality follows straight from the fact that \( \beta^c = SI(R_b) < SI(R_b) + \mu^*(R_b)SI^*(R_b) = \beta \). Intuitively, when the North is integrated with the lesser financially developed South, savings from the South flow into the credit market of the North, lowering the interest rate, hence increasing the incentive for Northern agents to chase higher returns from bubble speculation. Increased demand for the bubble naturally leads to an increased price in equilibrium.

The situation is slightly different but generally similar when \( n > 0 \). Recall from the closed economy that \( \beta^c \) is implicitly defined via the closed North’s savings-investment equation:

\[
SI \left( \frac{R_b}{1 - n\beta^c} \right) = \beta^c.
\]

The right panel of figure (4) represents \( \beta^c \) as the intersection of the dashed curve \( SI \) representing the left hand side and the 45 degree line representing the right hand side. On the other hand, in the open economy, \( \beta \) is implicitly defined via the *world* savings-investment equation (3.14):

\[
SI \left( \frac{R_b}{1 - n\beta^c} \right) + \left( \frac{\mathcal{A}^*(R_b)}{\mathcal{A}(\frac{R_b}{1 - n\beta^c})} \right) (1 - n\beta)SI^*(R_b) = \beta.
\]

The figure represents \( \beta \) as the intersection of the solid curve \( SI^* \) representing the left hand side and the 45 degree line. As in the closed model, as long as the \( SI^* \) is not as steep as the 45 degree line, then the \( SI^* \) curve and the 45 degree line will intersect in the positive quadrant if and only if the \( SI^* \) curve intersects the vertical line at a positive value, i.e., \( SI^*(R_b) > 0 \), or equivalently, \( R_b > R_{nb} \).

Furthermore, conditional \( R_{nb} < R_b \) so that the bubble exists, it is straightforward that \( \beta > \beta^c \). In the right panel of the figure, this can be seen by the fact that the solid line for \( SI^* \) lies above the dashed line \( SI \). Formally, because \( R_{nb}^c < R_{nb} < R_b \), it follows that \( SI^*(R_b) > 0 \). Therefore the \( SI \left( \frac{R_b}{1 - nx} \right) + \left( \frac{\mathcal{A}^*(R_b)}{\mathcal{A}(\frac{R_b}{1 - nx})} \right) (1 - nx)SI^*(R_b) > SI \left( \frac{R_b}{1 - nx} \right) \) for all
x. Combined with the equations that implicitly determine $\beta^c$ and $\beta$, we immediately get $\beta > \beta^c$.

The following result generalizes the argument above:

**Proposition 2.** [Integration facilitates bubbles] Assume parameters such that $R_{nc}^c < R_{nb}^c$ and the elasticity condition (2.30) holds for $0 < x < 2/(1 + n)$. Then:

1. There exists a BGP with a bubble in the North if and only if the world interest rate is low: $R_{nb} < R_b$. As a corollary, financial integration expands the parameter region in which a bubble can exist in the North: $\{R_{nc}^c < R_b\} \subset \{R_{nb} < R_b\}$.

2. Financial integration enlarges the bubble ratio: $\beta^c < \beta$.

**Proof.** Appendix.

We end the section by noting that an immediate corollary of the first claim in proposition 2 is that financial integration reduces the parameter region in which a bubble can exist in the South, i.e., $\{R_{nb} < R_b\} \subset \{R_{nc}^c < R_b\}$. The intuition is simple: financial integration with the more financially developed North raises the interest rate in the South, and as we already know, it is harder for bubbles to arise in higher interest rate environments.
4.2 Effects of bubble on capital flows

The previous section has shown that capital flows from the South to the North facilitates bubbles in the North. We now show a reverse effect: that a bubble in the North facilitates further South-to-North capital flows.

To account for capital flows, the trade balance is defined as savings minus total investment (in capital and in the bubble):

$$TB_t = S_t - (I_t + B_t).$$

Furthermore, we define the ratio of trade balance to net worth as

$$tb_t \equiv \frac{TB_t}{W_t + nB_t}.$$ 

On the bubble-less BGP, the trade balance ratio is simply the excess savings rate evaluated at the equilibrium world interest rate $R_{nb}$:

$$tb_{nb} = SI(R_{nb}),$$

and symmetrically for the South:

$$tb^*_{nb} = SI(R^*_{nb}).$$

On the BGP with a bubble in the North, the corresponding expression for the North is:

$$tb_b = SI\left(\frac{R_b}{1 - n\beta}\right) - \beta,$$

and the expression for the South is:

$$tb^*_b = SI^*(R_b).$$

It is straightforward to see that the bubble increases the trade balance ratio of the South. As the excess savings function $SI(R)$ is increasing in $R$, and as $R_{nb} < R_b$ (the condition for the existence of bubbles), it immediately follows that $tb^*_{nb} = SI^*(R_{nb}) < SI^*(R_b) = tb^*_b$.

On the other hand, when $n = 0$, the bubble also reduces the trade balance of the North. To see this, note that due to the world credit market clearing condition, the trade balance ratio of the North in the bubble BGP can be rewritten as the negative of the weighted trade balance ratio of the South:

$$-tb_b = \left(\frac{\mathcal{A}^*(R)}{\mathcal{A}(R)}\right)^{\frac{n}{\alpha}} tb^*_b.$$ 

It can be shown that for most standard productivity distributions, such as the log-normal dis-
tribution or the uniform distribution, the ratio \( \frac{A^*(R)}{\mathcal{A}(R)} \) is increasing in \( R \). Thus, \( \left( \frac{A^*(R_b)}{\mathcal{A}(R_b)} \right)^{\frac{1}{1-\alpha}} > \left( \frac{A^*(R_{nb})}{\mathcal{A}(R_{nb})} \right)^{\frac{1}{1-\alpha}} \). Combined with \( tb^*_b > tb^*_{nb} > 0 \), we get:

\[
\left( \frac{A^*(R_b)}{\mathcal{A}(R_b)} \right)^{\frac{1}{1-\alpha}} tb^*_b > \left( \frac{A^*(R_{nb})}{\mathcal{A}(R_{nb})} \right)^{\frac{1}{1-\alpha}} tb^*_{nb}.
\]

That is, not only the bubble raises the trade balance ratio of the South, it also raises the weighted trade balance ratio of the South. By using the world credit market clearing condition one more time, the right hand side of the inequality above is exactly equal to \(-t_{nb}\), the negative of the trade balance ratio of the North in the bubble-less BGP. Therefore, \(-tb^*_b > -tb^*_{nb}\), or equivalently,

\[tb^*_b > tb^*_{nb},\]

as desired. The argument is similar, although a bit more algebraically involved, when \( n > 0 \) but still satisfies the elasticity condition (2.30).

The following proposition generalizes the analysis above and formalizes the notion that bubbles in the North enhance the capital flows from South to North:

**Proposition 3.** [Bubble facilitates capital flows] Assume parameters such that \( R^c_{nb} < R^c_{nb} \), the elasticity condition (2.30) holds for \( 0 < x < 2/(1 + n) \), and \( \frac{A^*(R)}{\mathcal{A}(R)} \) is increasing in \( R \). Furthermore, assume \( R_{nb} < R_b \) so that the bubble BGP exists. Then the bubble increases trade deficit ratio of the North and the trade surplus ratio of the South:

\[tb^*_b < tb^*_{nb}\]

\[tb^*_b > tb^*_{nb}.

**Proof.** Appendix.

Note that the proposition immediately implies a corollary that the bubble reduces the trade-balance-over-GDP ratio of the North and increases that of the South. This is because, for the South, the inequality \( tb^*_b > tb^*_{nb} \) is equivalent to \( \frac{T_{b}^*}{Y_b} > \frac{T_{nb}^*}{Y_{nb}} \), as \( \frac{Y^*_b}{W^*_b} = \frac{1}{1-\alpha} = \frac{Y_{nb}}{W_{nb}} \). On the other hand, in the North, the inequality \( tb^*_b > tb^*_{nb} \) implies a weaker inequality \( \frac{T_{b}^*}{Y_b} < \frac{T_{nb}^*}{Y_{nb}} \). This is because \( \frac{Y_b}{W_{b+nb}} < \frac{Y_b}{W_b} = \frac{1}{1-\alpha} = \frac{Y_{nb}}{W_{nb}} \).

**Simulated example:** To illustrate the results on the effects of bubbles on business cycles, we conduct a numerical simulation. We stress that this exercise should not be interpreted as a quantitative analysis, but rather as an illustrative numerical example. In each of the four simulations, the global economy is in the bubble-less steady state in period \( t = 1 \). Bubbles
unexpectedly emerge in period \( t = 2 \) and eventually collapse in period \( t = 20 \). Figure 5 shows simulated equilibrium paths with a Northern bubble. It plots the North’s bubble ratio \( \beta_t \), aggregate output and aggregate consumption relative to their bubble-less steady state value \( y_t/y_{nb} \) and \( c_t/c_{nb} \), and the debt-to-output ratio \( d_t/y_t \). Note that the exogenous growth rate of \( g \) has been detrended from all variables.

As the figure shows, in these simulations, the bubble generally expands output, consumption and debt. After the collapses, the economy simply reverts to the bubble-less steady state. Note that the increase in debt and consumption is much more pronounced than the increase in output, which is qualitatively consistent with the U.S. experience during the 2000s: a large boom in housing prices that led to a big increase in debt and consumption, but a relatively mild increase in output.

Interestingly, figure 5 also shows that when the bubble collapses, aggregate consumption “overshoots,” i.e., drops sharply to a level below the bubble-less steady state value: \( c_{20} < c_{nb} \). Intuitively, this occurs if the bubble sufficiently increases the indebtedness of domestic agents, and its collapse then causes indebted old agents to deleverage by cutting down consumption. The contract in consumption is related to Eggertsson and Krugman (2012)’s result that deleveraging can depress aggregate demand and is qualitatively consistent with what happened during the Great Recession (e.g., Mian and Sufi 2010, 2014).

In summary, the figure illustrates that our model is consistent with the observed stylized features of bubble episodes, namely the boom and bust in output, consumption, debt and capital inflows.

![Fig. 5: Effects of a bubble episode on business cycles in the open North. All variables are detrended. GDP and aggregate consumption are reported relative to the bubble-less steady state.](image)

5 Conclusion

We have built a two-country open economy model with asset bubbles and global imbalances. The model provides an analysis of the underlying relationship between bubbles,
capital flows, and boom-busts in economic activities. Our results predict a close and rein-
forcing relationship between capital flows and asset bubbles. Specifically, the financial
integration of the South with the North leads to capital to flow into the North. Capital in-
flows in turn facilitate the emergence of large bubbles in the North, which further exacerbate
global imbalances. Several predictions of the model are qualitatively consistent with stylized
features of recent boom-bust episodes.

For tractability, we have made several simplifying assumptions, such as complete depre-
ciation of capital or risk-neutral agents who only consume in old age. However, we show
in the appendix through several extensions/robustness checks that these assumptions are
not essential and do not qualitatively affect our results. One potential direction for future
research is to extend our overlapping generation model to the Blanchard-Yaari perpetual
youth framework (Blanchard, 1985), which allows for a quantitative analysis that is outside
the scope of the current paper.

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