Saving-Constrained Households*

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Abstract

We present a set of facts on the joint dynamics of consumption and income that are difficult to reconcile with existing theory: 1) household-level consumption is as volatile as household income on average, 2) household-level consumption is relatively uncorrelated with income, 3) a large fraction of low-wealth households exhibit marginal propensities to consume near zero, and 4) lagged high expenditure is associated with low contemporaneous spending propensities. Our proposed interpretation of these facts is that household expenditure depends on time-varying minimum consumption thresholds that, if violated, yield substantial utility costs. We embed minimum consumption thresholds in an incomplete market heterogeneous-agent model and demonstrate that the minimum consumption thresholds are binding for many households. These households are “saving-constrained,” and they use additional income to save rather than spend.

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1 Introduction

In standard formulations of the Permanent Income Hypothesis, spending responds to shocks to income and credit access. Anecdotally, however, a large share of expenditure is autonomous and unanticipated. Peoples’ cars break down and they experience adverse health shocks, causing them to accumulate burdensome debt to cover their expenses. As a possible reflection of the prevalence of unexpected expenses, the Federal Reserve’s Report on Economic Well-Being explicitly asks households whether they could cover an unexpected $400 expense. Despite the prevalence of unanticipated expenses in policy debates (and the experiences of many households), they have yet to feature prominently in the literature on consumption. In this paper we propose a model that explicitly accounts for autonomous consumption in households’ consumption function. We begin by documenting a striking set of facts in the microdata on the joint dynamics of income and consumption.

First, for the average household in the Panel Study of Income Dynamics (PSID), consumption is as volatile as income. This household-level pattern stands in contrast to the smoothness of consumption that has been highlighted in the aggregate data (Campbell and Deaton (1989))). It might seem that such a pattern could be attributed to very high marginal propensities to consume (MPCs) out of income, either because many households are liquidity constrained (e.g., Kaplan and Violante (2014)) or because they perceive income changes to be permanent. Our second and third facts suggest that high spending propensities are unlikely to account for the first fact:

Our second fact is that for the average household, consumption and income are relatively independent. The average correlation in the data is only 0.2 whereas it is 0.6 in a standard calibrated heterogeneous-agent model. This suggests that explanations for volatile consumption cannot rely on a strong dependence of consumption on contemporaneous income. Facts 1 and 2 hold even when focusing exclusively on nondurable consumption, which suggests that durability of goods is not a driving force.

Our third fact is that many low-wealth/high-debt households exhibit MPCs near zero. This fact is primarily based on our review of the literature. A number of studies appear to find evidence of a large numbers of zero-MPC households, especially among the low-wealth households that are typically considered to be likely candidates to exhibit hand-to-mouth behavior. We highlight this evidence and replicate a prominent recent study of MPC heterogeneity, Misra and Surico (2014), to document the large fraction of low-wealth households with MPCs near zero.

Fourth, we document that contemporaneous spending propensities are strongly depen-
dent on prior expenditure. In particular, lagged high expenditure (on broad consumption or exclusively nondurables) is associated with low current MPCs out of income, conditional on wealth and other covariates. A household that experienced high consumption in the prior period exhibits a contemporaneous spending propensity that is approximately 20% lower than the average (unconditional) spending propensity. Each broad category of consumption contributes to high consumption episodes, roughly in proportion to each category’s average share of total consumption. Furthermore, the results are similar when examining consumption of nondurables, suggesting that durability is not driving the results. The notion that prior expenditure is inversely related (or related at all) to current MPCs is, at first glance, both counterintuitive and contrary to standard theory. If anything, modifications to standard theory would predict a positive relationship. Theories of habit formation, for example, would predict that high lagged expenditure should be associated with high desired current expenditure and therefore, if income is mean-reverting, a high contemporaneous MPC.

We propose a joint explanation of these facts based on a theory in which households face time-varying consumption thresholds that, if violated, yield substantial utility costs. These minimum consumption thresholds represent unanticipated shocks such as medical emergencies and auto repairs. When an adverse shock hits a household, it chooses to accumulate debt (reduce wealth) rather than let consumption fall below a threshold level. For example, rather than move out of a house or slash food consumption, households simply accumulate debt when faced with a large, unanticipated expenditure. Maintaining a low net asset position is optimal for these households but it is costly in the sense that, in the event of another adverse shock, households may be forced to consume below the threshold level (e.g., move out of a home or forgo medical care), which is associated with a large utility cost. Therefore, households for which consumption is against the minimum threshold use additional income to pay off debt (increase net assets) as a precautionary measure.

We refer to these households as saving-constrained households to capture the notion that, in the absence of binding minimum consumption thresholds, households would save rather than reduce their asset position. Minimum consumption thresholds effectively constrain households’ saving relative to a frictionless benchmark, just as credit constraints constrain borrowing relative to a frictionless benchmark.\(^1\)

We develop a heterogeneous-agent model featuring minimum consumption thresholds and calibrate it to match within-household dynamics of expenditure and income from the PSID. The model with time-varying minimum consumption thresholds easily fits the data, including the large within-household ratio of consumption volatility to income volatility (Fact 1), the low correlation between consumption and income (Fact 2), and the large share of house-

\(^1\)To see this mathematically, let \(c\) be consumption, \(y\) be income, \(a\) be current assets, and \(a'\) be assets carried to the next period, so the budget constraint is \(c + a' = y + a\). A minimum consumption constraint \(c \geq \bar{c}\) is equivalent to an upper bound on saving: \(a' - a \leq y - \bar{c}\).
holds with MPCs of zero (Fact 3). Furthermore, the model generates the path-dependence of spending propensities that we observe in the PSID (Fact 4). High lagged expenditure is associated with lower contemporaneous propensities to spend because high expenditure is a proxy for households being saving-constrained by high minimum consumption thresholds. Our framework is capable of explaining the key moments of the household-level relationship between consumption and income (including the prevalence of medium-to-low-income households with high savings propensities) with a straightforward, intuitive, and easy-to-implement modification to standard theory.\(^2\)

Our framework also helps to reconcile theory with otherwise puzzling consumer behavior documented by recent empirical work. In particular, Ganong and Noel (2019) show that, among low-wealth households receiving unemployment insurance (UI) benefits, spending drops precipitously upon the predictable expiration of UI benefits. Households in our model exhibit similar behavior when their consumption is at or below the minimum consumption threshold. In that case, a decline in income (even if anticipated) forces households to consume below the threshold and pay the utility cost.

The minimum consumption thresholds in our model are related to the notion of consumption commitments in Chetty and Szeidl (2007) – goods that are infrequently adjusted and are subject to transaction costs. In our theory, households subject to a shock (e.g., a medical emergency) by assumption do not adjust downward other components of consumption to maintain relatively stable expenditure. The notion that many aspects of consumption are difficult to adjust is one way to interpret our assumption that minimum consumption threshold shocks affect consumption broadly. In the face of a medical emergency, necessary education expense, or an auto repair, for example, households do not move out of their homes or downgrade household appliances. One can also interpret our minimum consumption thresholds as a reduced-form way to overlay stochastic maintenance costs on top of committed consumption. For example, the need for households to spend to repair their vehicles arises from the fact that the vehicle represents committed consumption. Even medical emergencies can be considered a form of maintenance, maintenance of a person’s physical body. The minimum consumption thresholds can also be interpreted as major expenses that Campbell and Hercowitz (2018) have documented are important for understanding household life-cycle saving behavior and the MPC distribution.

For comparison, we note here that alternative shocks will not replicate our facts. For example, many papers in the literature add shocks to discount factors in order to replicate the concentration of wealth observed in US data (Krusell and Smith (1998), Hubmer et al.

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\(^2\)Our framework shares with standard Bewley models the prediction that average MPCs in response to unanticipated transitory income shocks are low (< 0.2) compared to empirical evidence. We conjecture that integrating our theory of saving constraints with existing frameworks that deliver higher average MPCs (e.g. Kaplan and Violante (2014), Carroll et al. (2017)) will be fruitful for developing a comprehensive understanding of the MPC distribution.
While these shocks can help the model address Facts 1 and 2, they will not help with Fact 3 – in the stationary distribution poor households will have low discount factors and therefore high MPCs. Similarly, idiosyncratic variation in returns (Hubner et al. (2019)) will deliver the same prediction, as poor households will have low returns.

Our theory is likely to have important implications for the propagation of macroeconomic shocks and for the design of optimal social insurance policy. With respect to the propagation of macroeconomic shocks, in Miranda-Pinto et al. (2019) we show that saving constraints are important for understanding the cross-country relationship between fiscal effects on credit markets and consumer debt. In particular, higher debt (during periods of normal-to-loose credit supply) is associated with a larger share of saving-constrained households with low MPCs. Fiscal shocks relax credit markets more (increase interest rates less) in countries with high debt. A tractable general equilibrium extension of our model with minimum consumption constraints shows that the interest rate response to fiscal stimulus depends on consumer debt as in the data.

The remainder of the paper proceeds as follows. Section 2 lists the set of facts on the joint dynamics of income and consumption. Section 3 presents the model of saving constraints and implications for the MPC distribution. Section 4 evaluates the model’s fit to the data by comparing moments on the joint dynamics of income and consumption from simulated data to the moments from the PSID. Section 5 simulates the dynamic effects of one-time transfers to households in our model and demonstrates how the model can be reconciled with recent evidence on the effect of anticipated income declines. Section 6 concludes.

2 The Joint Dynamics of Household-Level Income and Consumption

Here we highlight evidence on household-level income and consumption that, when taken together, are difficult to reconcile with existing theories of heterogeneous households with uninsurable idiosyncratic income risk. The relevant facts are the following: (1) Consumption is as volatile as income for the average household. (2) Household-level consumption is relatively uncorrelated with income. (3) A large share of low-wealth households have spending propensities of effectively zero. (4) We present new evidence that lagged high expenditure is associated with low current spending propensities.

Our analysis with respect to Facts 1, 2, and 4 relies on data from the 1999-2017 Panel Study of Income Dynamics (PSID), which is a biennial panel study of households that are representative of the US population. The PSID is the most comprehensive dataset that tracks household-level expenditure and income over an extended period of time. Starting in 1999 the PSID began collecting data on a range of consumption categories, including
expenditures on health, housing, food, transportation, and education. In more recent waves the PSID added information on clothing and travel expenditures. With these additions, the PSID captures nearly all of the expenditure categories measured by the Consumer Expenditure Survey (CEX), which collects data at a higher frequency but does not have the longitudinal dimension that characterizes the PSID.\footnote{As discussed by Andreski et al. (2014), the consumption data in the PSID closely correspond to that from the CEX. A number of features of the PSID help to improve upon the accuracy of the responses relative to the CEX. For example, the PSID offers respondents unfolding brackets when they cannot recall the exact amount spent on the subcategories of expenditure. This approach both improves response rates and improves data accuracy. The PSID also collects information at a more aggregated subcategory level than does the CEX.} For our analysis, we normalize each variable (consumption, income, wealth) by the Personal Consumption Expenditure price index (PCE) for the year in which each measure is reported. To maintain a balanced panel, we restrict our sample to respondents that are in each wave of the PSID from 1999 through 2017. Our resulting sample consists of 6,159 households.

### 2.1 Facts 1 and 2: Consumption is Volatile and Relatively Independent of Income

Table 1 presents key moments from the joint dynamics of income and consumption in the PSID. For each household, we compute the standard deviations of the change in log consumption and the change in log income. We compute the within-household ratio of these standard deviations and report cross-sectional summary statistics: the mean, median, and standard deviation of $\frac{sd(d \log \text{Cons})}{sd(d \log \text{Inc})}$. We also compute the within-household correlation between the change in log consumption and the change in log income and report summary statistics. Finally, we estimate the autoregressive coefficient on log consumption.

To benchmark these consumption moments, we simulate data from a calibrated Bewley model. To calibrate the model, we simulate a quarterly income process and choose the parameters of the data generating process such that when the model is aggregated to a biennial frequency and truncated to 10 periods (the number of waves in our PSID sample), the estimated autoregressive process from the simulated data matches that in the data. Specifically, we assume that log income consists of a highly persistent component and a transitory component:

\[
y_{i,t} = x_{i,t} + z_{i,t} \\
x_{i,t} = \rho_x x_{i,t-1} + \sigma_x \varepsilon_{x,t} \\
z_{i,t} = \rho_z z_{i,t-1} + \sigma_z \varepsilon_{z,t},
\]

where $\rho_x = .99$, and $\varepsilon_x$ and $\varepsilon_z$ are i.i.d. $\mathcal{N}(0,1)$. In the PSID, however, we observe only log.
annual income \((a_{i,\tau}, \tau = 1, 2, 3, \ldots)\) sampled biennially. In the model, this corresponds to

\[
a_{i,1} = \log \sum_{t=1}^{4} \exp(y_{i,t})
\]

\[
a_{i,2} = \log \sum_{t=9}^{12} \exp(y_{i,t})
\]

\[
a_{i,3} = \log \sum_{t=17}^{20} \exp(y_{i,t})
\]

\[
\vdots
\]

In the PSID, we estimate the panel regression model (with household and time fixed effects)

\[
a_{i,\tau} = FI_i + \rho_a a_{i,\tau-1} + \gamma_{\tau} + \sigma_a \varepsilon_{a,\tau},
\]

which yields estimates of \(var(FI_i) \approx 0.87^2\), \(\rho_a \approx 0.09\), and \(\sigma_a \approx 0.86\). To calibrate the income process, we choose \(\sigma_x\), \(\sigma_z\), and \(\rho_z\) such that when we run the fixed-effects panel regression on model-simulated \(a_{i,\tau}\) for \(\tau \in [1, \ldots, 10]\), the resulting values for \(var(FI_i)\), \(\rho_a\), and \(\sigma_a\) match what we see in the PSID.\(^4\) We then use the calibrated income process to compute a standard Bewley model (one asset, heterogeneous agents, and uninsurable idiosyncratic income risk). The specifics of the model are outlined in Section 3 below.

Table 1 presents key moments from the joint dynamics of income and consumption in the PSID alongside the same moments from the calibrated Bewley model. The model statistics are computed based on quarterly simulations that are transformed into biennial data. It is readily apparent that consumption is more volatile (relative to income volatility) in the data than in the Bewley model (Fact 1). Both the average of \(sd(d\log Cons) / sd(d\log Inc)\) (1.05) and the median (0.85) far exceed the corresponding moments from the Bewley model. Furthermore, the ratio in the data exhibits a skewness that is not present in the data.

One possible explanation for volatile consumption in the data is that many households’ consumption tracks income due to very high MPCs. However, the second set of moments in Table 1 suggests that consumption is relatively independent of income (Fact 2). Whereas the average correlation between \(d\log Cons\) and \(d\log Inc\) in the model is 0.66, in the data it is only 0.24. Not only is consumption far less correlated with income than predicted by a standard model, it is also less persistent (bottom row of Table 1).

\(^4\)The relatively small time dimension in our panel allows for the possibility of Nickell (1981) bias in the estimate of the autoregressive coefficient. Our estimated coefficient is relatively low (0.15) and similar to estimates based on standard methods that address the bias (Anderson and Hsiao (1981) and Arellano and Bond (1991)). In calibrating the model we apply the same OLS estimator used in the data to the simulated data truncated to ten periods.
Joint dynamics of consumption and income: PSID and Bewley model

<table>
<thead>
<tr>
<th></th>
<th>PSID</th>
<th>Bewley model</th>
</tr>
</thead>
<tbody>
<tr>
<td>sd(d log Cons) / sd (d log Inc)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>1.05</td>
<td>0.49</td>
</tr>
<tr>
<td>Median</td>
<td>0.85</td>
<td>0.48</td>
</tr>
<tr>
<td>St Dev</td>
<td>0.84</td>
<td>0.07</td>
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<tr>
<td>corr(d log Cons, d log Inc)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.24</td>
<td>0.66</td>
</tr>
<tr>
<td>Median</td>
<td>0.27</td>
<td>0.66</td>
</tr>
<tr>
<td>St Dev</td>
<td>0.40</td>
<td>0.05</td>
</tr>
<tr>
<td>AR coefficient (Cons)</td>
<td>0.22</td>
<td>0.47</td>
</tr>
</tbody>
</table>

Note: The AR coefficient estimates from the PSID are based on an OLS regression with household and time fixed effects. Estimation procedures that account for possible Nickell bias yield slightly larger estimates. The PSID statistics are nearly identical when focusing exclusively on nondurable goods.

The summary statistics from the PSID in Table 1 are nearly identical when focusing exclusively on non-durable goods (aggregate consumption net of purchases of new cars, furniture, appliances, and other household items such as floor coverings). Therefore, durability alone cannot account for consumption that is volatile and relatively independent of income.

2.2 Fact 3: Many Low-wealth Households Have MPCs Near Zero

Here we discuss evidence that many low-wealth households have MPCs of zero. We first survey and synthesize recent work that documents zero-MPC households. We then extend a prominent study of MPC heterogeneity, Misra and Surico (2014), to offer additional details on the prevalence of low MPCs among low-wealth households.

Much of the recent theoretical work on MPCs has focused on rationalizing the existence of high-MPC households. For example, Kaplan and Violante (2014) and Carroll et al. (2017) show that carefully calibrated models with liquidity constraints and preferences exhibiting prudence can yield households with reasonably high MPCs through (i) precautionary savings, which generate a steep consumption function for lower wealth individuals, and (ii) binding liquidity constraints, which render some households “hand-to-mouth.”

To date there has been limited theoretical work that addresses an otherwise elusive set of findings from the empirical literature: a substantial fraction of households have an MPC of zero, and many of these households are not wealthy or high-income.

Most recently, Bunn et al. (2018) report an MPC of zero for 77% of surveyed British households with respect to positive shocks. Furthermore, the probability of reporting an
MPC of zero is significantly higher for households with a mortgage loan-to-value of ratio of 75–90% (vs. less than 75%). Prior studies have likewise documented a substantial (but smaller) share of households with MPCs near zero. Using the 2010 Italian Survey of Household Income and Wealth, Jappelli and Pistaferri (2014) find that around 22% of households would have an MPC of zero in response to a hypothetical income shock equal to a typical month of earnings. While the fraction of households with an MPC of zero increases in cash-on-hand percentile, still around 10% of the most cash poor households have a zero MPC.

Shapiro and Slemrod (2003) and Sahm et al. (2015) survey US households around tax changes and present evidence related to zero-MPC behavior. Instead of soliciting precise MPCs, they ask households whether take cuts (hikes) lead them “mostly” to increase (decrease) spending, mostly to increase (decrease) saving, or mostly to pay off (pay off less) debt. With respect to the Economic Growth and Tax Relief Reconciliation Act of 2001 (EGTRRA) Shapiro and Slemrod (2003) report that 78.2% of households say they would mostly increase saving or pay off debt. The authors find higher saving/delevering response rates for lower income households (vs. higher income households), non-stockholders (vs. stockholders), and households with small amounts of stock (vs. households with more stock). They also show households that “use credit” to “pay for unexpected expense” have higher saving/delevering rates than ones that “use savings” or “cut back spending.” In the Sahm et al. (2015) 2013 retrospective survey concerning the 2011-2012 payroll tax holiday, 65% of households say they mostly adjusted saving and debt/borrowing (the corresponding number was 86% in the prospective survey from March/April 2011). In the retrospective survey, more than 50% of these saving/debt adjusters have household income less then $75,000 and around 20% make less than $35,000.

Much of the above evidence is based on surveys of what households did or would do in response to an income shock. Misra and Surico (2014) estimate MPC heterogeneity based on observed responses to transfer shocks. Using the Consumer Expenditure Survey (CEX) to study both EGTRRA and the Economic Stimulus Act of 2008 (ESA), their quantile regressions imply that for 40–50% of households the spending response to tax rebates is not statistically different from zero. Inspection of Figures 1 and 4 in of Misra and Surico (2014) suggests many of these households have moderate-to-low-income (less than around $50,000).

Our survey of the existing evidence suggests that many low-wealth, high-debt households (the type of households typically associated with high MPCs) exhibit MPCs near zero. This echoes the survey of the literature in Carroll et al. (2017), who write, “much of the empirical work does not find that the consumption response of low-wealth or liquidity constrained households is statistically higher . . .”. It is also consistent with Kueng (2018), who finds that average MPCs are higher among households with higher income.

To obtain a more complete understanding of the prevalence of low MPCs among low-
wealth households, we replicate the Misra and Surico (2014) study of MPC heterogeneity. Their quantile regression approach estimates MPCs for subgroups of households, where the subgroups are based on household quantiles of consumption changes during the period of a tax rebate. With their quantile estimates in hand, we then examine the relationship between incidence of near-zero MPC and household wealth.

2.2.1 Short-Term Spending Propensities from the 2001 Tax Rebate

The mailing of the 2001 tax rebate was randomized based on the penultimate number of the tax filer social security number. Hence, the rebate receipt was exogenous to individual characteristics. Using households level data on consumption from the CEX and individual tax records, Johnson et al. (2006) estimate the short-term consumption responses to the tax rebate receipt. The main specification in Johnson et al. (2006) is

\[ \Delta C_{it+1} = \sum_s \alpha_0s \times M_s + \alpha_1' X_{it} + \alpha_2 R_{it+1} + u_{it+1}, \]

where \( \Delta C_{it+1} \) is household \( i \)'s change in nondurable consumption in the three month period when the tax rebate was received. \( M_s \) is a set of time controls that capture seasonal effects and aggregate shocks. The matrix \( X_{it} \) contains household controls, in particular average age and the change in the number of family members. The main variable \( R_{it+1} \) is the total dollar amount of tax rebate received by households \( i \) in the three months period \( t+1 \).

Misra and Surico (2014) amend the approach in Johnson et al. (2006) to account for the possibility that consumption responses may be heterogeneous, even within subgroups based on income. The authors estimate a version of equation (1) using quantile regression and find that high-income households are likely to have very low and very high consumption responses to the 2001 (and 2008) tax rebate.

\[ \Delta C_{it+1} = q(R_{it+1}, X_{it}, M_s, \lambda_{it+1}) \quad \text{with} \quad \lambda_{it+1}|R_{it+1}, X_{it}, M_s \sim U(0,1), \]

where \( \lambda_{it+1} \) captures the unobserved heterogeneity in households with similar observed characteristics \( (R_{it+1}, X_{it}, M_s) \). Let \( q(R_{it+1}, X_{it}, M_s, \tau) \) be the conditional \( \tau \)-th quantile of \( \Delta C_{it+1} \), given observables. For each \( \tau \in (0,1) \), the linear quantile model is

\[ \Delta C_{it+1} = q(R_{it+1}, X_{it}, M_s, \tau) = \sum_s \alpha_0s(\tau) \times M_s + \alpha_1(\tau)' X_{it} + \alpha_2(\tau) R_{it+1}. \]

The estimated consumption responses are common within a quantile \( \tau \) but are heterogeneous across quantiles, representing unobserved heterogeneity.
2.2.2 Extending the Misra and Surico (2014) Analysis

We start by replicating the estimation of $\alpha_2$ in equation (3) in Misra and Surico (2014). We focus on the estimated tax rebate coefficient of non-durable consumption for the 2001 tax rebate. Then, we gather additional information from the CEX on households’ wealth. We define wealth as the sum of the value of holdings of checking accounts, saving accounts, U.S. bonds, stocks, and property, minus the outstanding mortgage debt and outstanding non-mortgage debt. Non-mortgage debt is composed of credit card debt, bank loans, credit union debt, and dentist and hospital debt.

Figure 1 shows the distribution of MPCs for the group of low-wealth households (below median wealth). Some low-wealth households exhibit very large ($\approx 1$) MPCs, but over 40% of low-wealth households have an MPC estimate of zero or below.\(^5\)

In the Misra and Surico (2014) analysis, MPC estimates differ among households with the same wealth but are the same for households in the same consumption quantile. In Figure 2 we plot, for each quantile, the MPC estimate (with standard error bands) along with median wealth for households in that quantile. Of the twenty quantiles, eight have MPC estimates below zero (and are classified in Figure 2 as zero-MPC). Notably, the median wealth for most of these quantiles is low, but the lowest-wealth quantiles have higher MPC estimates (between 0.1 and 0.5). Further, three of the zero-MPC quantiles have moderate levels of median wealth (between $45,000 and $65,000). Therefore, zero-MPC behavior appears to be prominent among the low-but-not-lowest wealth levels and appears even at moderate wealth levels.

![Figure 1](image.png)

Note: This figure plots the distribution of estimated MPCs for U.S Households with wealth below the median. We classify below-zero MPC estimates as having an MPC of zero.

\(^5\)We classify MPC estimates below zero as zero.
2.3 Fact 4: Lagged High Expenditure is Associated with Low Contemporaneous Spending Propensities

In the standard heterogeneous agent model with uninsurable idiosyncratic income risk, current assets and income are sufficient information to infer households’ optimal consumption decisions. Therefore, lagged expenditure contains no additional relevant information for determining agents’ consumption. Here we show that lagged expenditure indeed contains additional relevant information, and in particular that lagged high expenditure is associated with low contemporaneous propensities to spend out of additional income.

Our first step is to estimate spending propensities out of additional income. We then identify episodes of prior high consumption in the past to determine whether prior high expenditure is associated with differential contemporaneous spending propensities. We identify a household as experiencing high expenditure when its expenditure exceeds its within-household average by a standard deviation. Our baseline specification is

$$\log C_{it} = \beta_0 \log I_{it} + \beta_1 HighC_{it-1} + \beta_2 \log I_{it} \times HighC_{it-1} + \gamma X_{it} + \epsilon_{it},$$  \hspace{1cm} (4)$$

where $HighC_{it-1}$ is a dummy variable that equals one when the expenditure of household $i$ exceeds the within-household average by a standard deviation at period $t - 1$. $X_{it}$ includes

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6One limitation of the PSID is that it is impossible to distinguish between anticipated and unanticipated changes in income. The CEX, on the other hand, can be merged with information on tax rebates to estimate marginal spending propensities. However, the CEX does not have the unique panel structure that the PSID contains and hence is uninformative for the relationship between lagged expenditure and current spending propensities. The implication of potential anticipation effects in the PSID is that estimated contemporaneous spending propensities out of observed income changes should be lower than spending propensities out of unanticipated income changes.
log(wealth), the interaction of the wealth term with log(income), and a quadratic in age of the head of the household. We also control for household and time fixed effects.

Table 2 shows that the elasticity of expenditure with respect to income is 0.16 (column (1)). Column (2) shows this spending propensity varies with lagged high expenditure, and in particular that having high lagged expenditure reduces the spending propensity by 0.030, or 18.4% of the average effect (0.163) of income, indicating a large state-dependence of spending propensities. In a standard theoretical framework, beginning-of-period wealth subsumes any information conveyed by prior consumption, so it is possible that the heterogeneous effect associated with lagged expenditure simply reflects spending propensities that vary by wealth. To address this possibility, in column (3) we include then interaction of (log) wealth and current (log) income. The coefficient on the interaction of income with lagged high expenditure is slightly attenuated but remains economically and statistically significant.

Our main specification is presented in column (4). Here, we isolate incidents of high lagged expenditure that are not associated with high income. The specific reason for doing so will be apparent from the theory in Section 3. Briefly, in standard heterogeneous agent models high consumption is caused by high income and is not associated with lower future spending propensities. In the model we develop below high consumption can also arise for reasons unrelated to contemporaneous income, and it is these idiosyncratic consumption episodes that cause lower future spending propensities. Therefore, to isolate the mechanism that we will propose, it is important to isolate episodes of high consumption that are not associated with high contemporaneous income. We classify these episodes as “high expenditure episodes.” In particular, a high expenditure episode is a dummy variable that equals unity when household consumption exceeds its within-household mean by a standard deviation and household income does not exceed its within-household mean by a standard deviation. These episodes capture periods of high consumption that are not driven by high contemporaneous income (see Table 3 for a summary of the indicator variables and their precise definitions). Column (4) of Table 2 replaces the indicator for high expenditure with the indicator for a high expenditure episode. Here, the interaction term is even more negative: a high expenditure episode is associated with a spending elasticity that is lower by 0.039, or 23.9% of the average elasticity of spending with respect to income. High expenditure that is not associated with high contemporaneous income is associated with substantially lower future spending propensities.

The negative association of high consumption is unique to episodes with low income. When we replace the indicator for a high consumption episode with an indicator for high consumption and high income, the effect on future spending propensities flips signs (column (5)). Therefore, the negative relationship between high expenditure and future low spending propensities.

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7 Estimates of the coefficient on the interaction term based on the Anderson-Hsiao procedure and the Arellano-Bond procedure are -0.025 and -0.035, respectively.
Table 2  
Lagged Expenditure and Spending Propensities in the PSID.

<table>
<thead>
<tr>
<th>Dependent variable: log($C_t$)</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>log($I_t$)</td>
<td>0.163***</td>
<td>0.170***</td>
<td>0.181***</td>
<td>0.183***</td>
<td>0.168***</td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td>(0.009)</td>
<td>(0.014)</td>
<td>(0.014)</td>
<td>(0.014)</td>
</tr>
<tr>
<td>High $C_{t-1}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.409***</td>
<td>0.308**</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.121)</td>
<td>(0.113)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>log($I_t$) · High $C_{t-1}$</td>
<td>-0.030***</td>
<td>-0.021**</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.011)</td>
<td>(0.010)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>High $C_{t-1}$ &amp; not high $I_t$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>log($I_t$) · (High $C_{t-1}$ &amp; not high $I_t$)</td>
<td>-0.039***</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.011)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>High $C_{t-1}$ &amp; high $I_{t-1}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>log($I_t$) · (High $C_{t-1}$ &amp; high $I_{t-1}$)</td>
<td>0.122***</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.011)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Control for log(wealth) X log($I_t$)</td>
<td>NO</td>
<td>NO</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>N</td>
<td>61590</td>
<td>55431</td>
<td>55431</td>
<td>55431</td>
<td>55431</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.65</td>
<td>0.66</td>
<td>0.67</td>
<td>0.67</td>
<td>0.69</td>
</tr>
</tbody>
</table>

Note: A household has high income, High $I_t$, (expenditure, High $C_t$) in periods in which income (expenditure) is over a standard deviation above average income (expenditure) for the household. A high expenditure episode (High $C_{t-1}$ not high $I_t$) is a dummy that equals unity when a household experiences high expenditure but not high income. All regressions control for time and household fixed effects, log(wealth), and a quadratic in age of the head of household. Robust standard errors in parentheses. ***p<0.01, ** p<0.05, *, p<0.1.

propensities is driven by episodes in which high expenditure is not associated with high income.

Decomposing High Expenditure Episodes. Which categories of expenditure drive high expenditure episodes (henceforth referred to as “episodes”)? Are episodes primarily driven by subsets of expenditure, or do all components of expenditure contribute to these episodes? To address these questions, we first examine how much consumers spend on different categories during episodes relative to average spending on each category. Column (1) of Table 4 shows average (across households) expenditure shares for each category of expenditure, where the categories correspond to PSID classification schemes. Column (2) shows the expenditure shares during episodes. Expenditure shares during episodes (column (2)) are generally

---

8To compute the statistics in column (2), we first demean category-specific expenditure for each household.
Table 3
Indicator Variable Definitions and Incidence

<table>
<thead>
<tr>
<th>Indicator Variable</th>
<th>Description</th>
<th>Incidence</th>
</tr>
</thead>
<tbody>
<tr>
<td>High Expenditure</td>
<td>$Expenditure_t &gt; \text{Mean}(Expenditure_t) + SD(Expenditure_t)$</td>
<td>0.16</td>
</tr>
<tr>
<td>High Income</td>
<td>$Income_t &gt; \text{Mean}(Income_t) + SD(Income_t)$</td>
<td>0.26</td>
</tr>
<tr>
<td>High Expenditure Episode</td>
<td>High Expenditure and Not High Income</td>
<td>0.10</td>
</tr>
<tr>
<td>High Expenditure &amp; High Income</td>
<td>High Expenditure and High Income</td>
<td>0.07</td>
</tr>
</tbody>
</table>

similar to average expenditure shares (column (1)). Two categories are noticeably more prevalent during episodes: education and transportation. Food and housing are less prevalent during episodes.

Table 4
Decomposing High Expenditure Episodes

<table>
<thead>
<tr>
<th></th>
<th>Share of Total Expenditure (1)</th>
<th>Ratio of Category Expenditure Relative to Total Expenditure during High Expenditure Episodes (2)</th>
<th>Coefficient from Linear Probability Model (3)</th>
<th>Coefficient from Probit Model (4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food</td>
<td>0.18</td>
<td>0.07</td>
<td>0.14</td>
<td>0.87</td>
</tr>
<tr>
<td>Housing</td>
<td>0.44</td>
<td>0.36</td>
<td>0.31</td>
<td>1.60</td>
</tr>
<tr>
<td>Transportation</td>
<td>0.27</td>
<td>0.37</td>
<td>0.39</td>
<td>1.84</td>
</tr>
<tr>
<td>Education</td>
<td>0.04</td>
<td>0.10</td>
<td>0.14</td>
<td>0.92</td>
</tr>
<tr>
<td>Child Care</td>
<td>0.01</td>
<td>0.01</td>
<td>0.05</td>
<td>0.36</td>
</tr>
<tr>
<td>Health</td>
<td>0.08</td>
<td>0.07</td>
<td>0.11</td>
<td>0.75</td>
</tr>
</tbody>
</table>

Note: This table presents statistics for the broad categories that make up the measure of total expenditure. Expenditure on Clothing, Trips, and Other Recreation are not included in the measure of total expenditure since they were only recorded beginning in 2005. In column (2), expenditure during episodes is relative to within-household averages. In columns (3) and (4), the depicted statistics are the coefficients from a regression of an indicator for a high expenditure episode on indicator variables for high sub-category expenditure. All regression coefficients are significant at the p<0.01 level.

Columns (3) and (4) offer an alternative approach to examining the relevance of different expenditure categories in driving episodes. Here, we identify episodes for each category and examine the extent to which category-specific episodes predict aggregate expenditure episodes. For example, the dummy for a transportation episode is set to unity whenever a household’s transportation expenditure exceeds its within-household average by a standard deviation (and income is not high). We then regress the dummy for an episode on indicators for the category-specific episodes. The pattern that emerges from both OLS (column (3)) and probit (column (4)) models is that high sub-category expenditure is associated with a high-expenditure episode for each category of expenditure. Furthermore, the likelihood that to obtain a measure of excess expenditure at any point in time. We then average over households experiencing an episode to obtain average excess expenditure for a category during episodes. We then do the same for total expenditure and take the ratio of the two.
any given category-specific episode is associated with a total expenditure episode is broadly proportional to that category’s share of total expenditure and follows a similar pattern of relevance that is depicted in column (2).

**Alternative specifications.** Here we extend the analysis in Table 2 to examine different measures of income and expenditure. First, to help isolate the unanticipated component of income, we first replace the continuous measure of income in the regressions with an indicator for high income (defined above). Under the assumption that abnormal realizations of income are less likely to be anticipated, the indicator variable is more likely to isolate unanticipated changes in income.\(^9\) We also examine an alternative measure of expenditure that excludes purchases of durables. Specifically, we identify high expenditure episodes based on extreme realizations of expenditure net of purchases of automobiles, furniture, appliances, and other household items such as floor coverings. This will help guide the theory that we develop to explain the joint dynamics of expenditure and income.

**Table 5**

<table>
<thead>
<tr>
<th>Specification with Alternative Expenditure and Income Measures</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dependent variable: log(expenditure)</td>
</tr>
<tr>
<td>Income measure: log(income)</td>
</tr>
<tr>
<td>Expenditure measure used to identify high-expenditure episodes:</td>
</tr>
<tr>
<td>All Nondurables</td>
</tr>
<tr>
<td>(1) (2)</td>
</tr>
<tr>
<td>Coefficient on Interaction term</td>
</tr>
<tr>
<td>-0.039***</td>
</tr>
<tr>
<td>(0.011)</td>
</tr>
<tr>
<td>Fraction of average effect of income measure</td>
</tr>
<tr>
<td>-0.24</td>
</tr>
<tr>
<td>Note: Nondurable expenditure is defined as total expenditure net of purchases of automobiles and furniture. An episode is defined as a period in which a household has high expenditure but not high income. A household has high expenditure (income) in periods in which expenditure (income) is over a standard deviation above average expenditure (income) for the household. All regressions include the income measure, the episode indicator, a quadratic in age, log(wealth) and its interaction with the income measure, and time and household fixed effects. The table reports only the coefficient on the interaction between lagged episode and the income measure. Robust Standard Errors in parentheses. **p&lt;0.01, * p&lt;0.05, *, p&lt;0.1.</td>
</tr>
</tbody>
</table>

According to the results in Table 5, replacing the income measure with an indicator for high income produces a similar pattern: the propensity to spend in response to a high income realization is lower in the presence of a lagged high expenditure episode (columns (1) and (3)). If anything, the magnitude of the negative interaction term is larger as a fraction of the average effect of high income. Furthermore, the negative effect of lagged high expenditure on spending propensities is just as strong when limiting the expenditure

\(^9\)Note that anticipation effects should, if anything, reduce our coefficient estimates toward zero. The fact that spending propensities are positive suggests that, in the absence of credit constraints, households behave as if changes in income are to some extent unanticipated.
measure used to identify episodes to nondurables (columns (2) and (4)). This indicates that modeling durable goods is not necessary for understanding the effect of lagged expenditure on spending propensities.

2.4 Summary and Interpretation

Facts 1 through 4 are puzzling from the perspective of standard heterogeneous-agent models with idiosyncratic income risk. It might seem that volatile consumption (Fact 1) could be attributed to households having higher MPCs than in the benchmark Bewley model. But a model featuring a stronger effect of income on consumption would exhibit a higher correlation between consumption and income, contrary to Fact 2. It would also not help resolve Facts 3 and 4.

It is possible that measurement error in consumption contributes to consumption that is volatile and independent of income (Facts 1 and 2). But significant measurement error in consumption would also prevent us from detecting Fact 4 by attenuating toward zero the estimate of the coefficient on the interaction between lag high consumption and contemporaneous income. Nor can measurement error in the PSID account for the existence of low-wealth, low-MPC households (Fact 3). Therefore, we conclude that a joint explanation of the facts is not readily apparent, and we turn to a theory of saving constraints to help rationalize the evidence.

3 A Theory of Saving-Constrained Households

Here we present a theory to rationalize the evidence presented above. We introduce random minimum consumption thresholds into a standard Bewley model with capital and calibrate it to match the consumption and income dynamics from the PSID.\textsuperscript{10} These minimum consumption thresholds represent stochastic maintenance costs for aspects of current consumption that are determined by prior decisions and costly to adjust in the short-term (“consumption commitments” –Chetty and Szeidl (2007)). For example, automobiles (committed consumption) may break down and require repairs.

In the ergodic distribution of the model, many households have near-zero MPCs. Average MPCs are U-shaped in wealth (so low MPCs are concentrated in the middle of the bottom half of the wealth distribution), and the higher MPCs arise both from low-wealth households violating the minimum consumption threshold and from high-wealth households with little chance of hitting the threshold. In Section 4 we take the model to our puzzling facts from Section 2, demonstrating that the model generates the dependence of spending propensities

\textsuperscript{10}Specially, we use the evidence in Facts 1 through 3 as calibration targets. We do not target the regression evidence from Fact 4.
on lagged high expenditure, the high volatility of consumption relatively unrelated to income, and the low persistence of consumption (in addition to the existence of many low-wealth, zero-MPC households).

3.1 Model

The economy consists of a measure one of infinitely-lived households that are ex ante identical and a representative firm that hires capital and labor to produce the single tradable consumption good. The households and firm participate in a global capital market with exogenous interest/rental rate $r$. The labor market, in which the firm hires household labor at wage rate $w$ is, however, purely domestic. The recursive problem of a household is

$$V(k, z, x, c) = \max_{c \geq 0, k' \geq b} \left\{ \log(c) - \lambda \max\{c - c, 0\} + \beta E_{z', x', c'} \left[V(k', z', x', c') \mid z, x, c\right] \right\}$$

subject to the budget constraint

$$c + k' \leq (1 + r - \delta + \phi 1(k \leq 0)) k + w \exp(z + x) h,$$

where $V$ is the value function, $c$ is consumption (the numeraire), $k$ is capital wealth (which exogenously depreciates at rate $\delta \geq 0$), $z$ is persistent idiosyncratic household productivity, $x$ is (nearly) permanent idiosyncratic productivity, and $c$ is a persistent minimum consumption threshold. If the household consumes less than $c$, it must pay utility cost $\lambda(c - c)$ for some $\lambda \geq 0$. Borrowing ($k < 0$) entails a cost $\phi(-k)$, for some $\phi \geq 0$, so the household interest rate on borrowing is higher than the interest rate on saving. In each period, the household inelastically supplies effective labor $\exp(z + x) h$ at wage $w$. For any variable $q$, $q'$ represents its value in the subsequent period.

We assume that household productivity and minimum consumption thresholds evolve according to:

$$z' = \rho_z z + \epsilon_z'$$
$$x' = \rho_x x + \epsilon_x'$$
$$c' = (1 - \rho_c) c + \rho_c c + \epsilon_c', $$

where $\epsilon_j, j \in \{z, x, c\}$, is an idiosyncratic mean-zero shock with standard deviation $\sigma_j$. Define the stationary aggregate labor supply to be $\overline{H}$.

In each period, the representative firm chooses capital $K$ and effective labor $L$ to solve

$$\max_{K,L} \left\{ K^\alpha L^{1-\alpha} - r K - w L \right\}.$$

We examine stationary equilibria, which are denoted by stars and consist of constant firm
capital $K^*$ and labor $L^*$, a constant wage $w^*$, a constant household wealth distribution $\Omega^*$, and household value and policy functions $V^*$, $c^*$, and $k^{*}$ such that (1) the value and policy functions solve the household problem given prices, (2) $K^*$ and $L^*$ solve the firm problem:

$$r = \alpha (L^*/K^*)^{1-\alpha}, \quad w^* = (1-\alpha)(K^*/L^*)^\alpha = (1-\alpha)(r/\alpha)^{\frac{\alpha}{\alpha-1}},$$

(3) the labor market clears: $L^* = \overline{H}$, and (4) $\Omega^*$ is generated by $k^*$. Let $K$ denote aggregate household capital, define $y = (r - \delta + \phi 1(k \leq 0))k + w \exp(z + x)\overline{h}$ to be household income, and let $Y$ be aggregate household income.

### 3.2 Calibration

We assume a period is one quarter and calibrate the model with a two-step procedure. First, we choose the productivity process parameters to approximate household income from the PSID (as discussed in Section 2) and set the borrowing cost, capital share, depreciation rate, global interest rate, and borrowing constraint to reasonable values: $\phi = 0.03$, $\alpha = 0.36$, $\delta = 0.0125$, $r = 0.0225$, $b = -1$, $\rho_z = 0.63$, $\sigma_z = 0.79$, $\rho_x = 0.99$, and $\sigma_x = 0.14$. In the second step, we choose the remaining parameters (discount rate, utility cost, and $\xi$ process) to target the following moments from the ergodic distribution of the stationary equilibrium:\footnote{Given parameters, we use Rouwenhorst’s method to discretize the productivity and $\xi$ processes as Markov processes, three states for each productivity process and seven states for the expenditure shock. Given exogenous $r$ and aggregate labor supply $\overline{H}$, firm capital and the equilibrium wage follow trivially from firm optimality. We then solve the household problem with standard global methods, yielding the household policy functions and the stationary wealth distribution.}

\[
\begin{align*}
\frac{K}{Y} &= 12 \\
\text{mean}(\text{Corr}(d\log y, d\log c)) &= 0.2 \\
\text{mean}\left(\frac{\text{Std}(d\log c)}{\text{Std}(d\log y)}\right) &= 1.04 \\
\text{Fraction}(k < 0) &= 0.1 \\
\text{Corr}(\log(c_t), \log(c_{t-1})) &= 0.25 \\
\text{Fraction}(|c - \xi| < 0.01) &= 0.3 \\
\text{mean}(MPC) &= 0.2,
\end{align*}
\]

where $\text{Corr}(d\log y, d\log c)$, $\frac{\text{Std}(d\log c)}{\text{Std}(d\log y)}$, and $\text{Corr}(\log(c_t), \log(c_{t-1}))$ are computed at the biennial frequency and are based on the values in Table 1 above. Via global optimization, the best–fit parameter values are $\beta = 1/1.025$, $\lambda = 5.014$, $\rho_c = 0.65$, $\sigma_c = 1.48$, and $\mu_c = 0.0047$. The baseline Bewley model is computed analogously but without the $\xi$ process and with $\beta$ re-calibrated to match $K/Y = 12$.\footnote{Given parameters, we use Rouwenhorst’s method to discretize the productivity and $\xi$ processes as Markov processes, three states for each productivity process and seven states for the expenditure shock. Given exogenous $r$ and aggregate labor supply $\overline{H}$, firm capital and the equilibrium wage follow trivially from firm optimality. We then solve the household problem with standard global methods, yielding the household policy functions and the stationary wealth distribution.}
3.3 Results

The zero-MPC households are evident from the consumption functions of households with different realizations of $c$. Figure 3 shows that households with higher values of $c$ tend to have consumption that is flat with respect to wealth (MPCs of zero) until wealth is sufficiently high that the minimum consumption threshold is no longer binding. These constrained households use all additional wealth/income to save. However, not all low-wealth households with high realizations of $c$ have MPCs of zero. The poorest households cannot even achieve the minimum consumption level. They experience severe dis-utility and consume all additional income, up until they are able to consume at the threshold.

![Figure 3](image)

Note: The left column shows the model consumption policy functions at different income levels, and the right column shows the steady-state wealth distribution conditional on these income levels. Line thickness corresponds to the value of $c$. Low (Middle, High) income means both the permanent and persistent components of productivity are at their low (middle, high) discretized values.

What does this imply for the MPC distribution across households? Comparing the wealth distributions in Figure 3 with the consumption functions, we see that in the ergodic distribution many agents have an MPC of zero and the higher MPCs come from the very rich and poor by wealth: the former have moderate MPCs consistent with the Permanent Income Hypothesis, while the latter are violating their threshold and anxious to not pay the utility
cost. Figure 4 is the ergodic MPC histogram for below-median wealth households. Echoing the corresponding Figure 1 from the data, Figure 4 shows that for low-wealth households, the largest mass of MPC is around zero. Relative to the data, the model is lacking intermediate MPC low-wealth households, although both the model and data histograms exhibit a mass of very high MPC households.

Figure 4
Note: The figure shows the model’s ergodic MPC distribution for households with below-median wealth. The bin size is 0.0219.

Figure 5 shows the model’s average MPCs by wealth decile, illustrating that MPCs are U-shaped in wealth. The lowest MPCs households are, on average, not rich. They are poor enough for the minimum consumption level to matter but not so poor that they violate it. This pattern is reminiscent of Misra and Surico (2014), who uncover a non-monotonic U-shaped relationship between either income, home ownership (with a mortgage), or liquid assets and spending propensities. They write, “the largest propensity to consume out of the tax rebate tends to be found for households with both high levels of mortgage debt and high levels of income.”

4 Model Fit: Comparison to PSID

As discussed above, the model features a substantial fraction of zero-MPC households that have low-to-median wealth (Fact 3). Here we examine the model’s ability to improve the fit
to the PSID with respect to Facts 1, 2, and 4. To make the results comparable with the PSID, we transform the simulated quarterly data into biennial data.

With respect to Facts 1 and 2, Table 6 shows that including the minimum consumption shock substantially improves the fit to the PSID. Whereas consumption in the standard Bewley model is insufficiently volatile, too correlated with income, and too persistent, in the saving constraint model consumption is nearly as volatile as income (on average), relatively uncorrelated with income, and less persistent. Consumption volatility in the saving constraint model also exhibits skewness as in the PSID.

With respect to Fact 4, we first demonstrate that the theory fits the empirical relationship between lagged expenditure and spending propensities at the biennial frequency. We then analyze simulated quarterly data (for which we can directly observe saving constraints) to assess how well the empirical specification captures the behavior of saving constrained households. In assessing the model’s fit to Fact 4, it is important to note that the regression evidence was not used to calibrate the model.

Table 7 shows that spending propensities out of income are lower when households experienced high expenditure episodes in the previous period (column (4)). In the Bewley model without minimum consumption thresholds, average spending propensities are approximately independent of lagged high expenditure (column (6)).
### Table 6
Consumption and Income Moments

<table>
<thead>
<tr>
<th></th>
<th>PSID</th>
<th>Bewley model</th>
<th>Saving Constraint Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>sd(d log Cons) / sd (d log Inc)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>1.05</td>
<td>0.49</td>
<td>0.90</td>
</tr>
<tr>
<td>Median</td>
<td>0.85</td>
<td>0.48</td>
<td>0.39</td>
</tr>
<tr>
<td>St Dev</td>
<td>0.84</td>
<td>0.07</td>
<td>0.91</td>
</tr>
<tr>
<td>corr(d log Cons,d log Inc)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.24</td>
<td>0.66</td>
<td>0.34</td>
</tr>
<tr>
<td>Median</td>
<td>0.27</td>
<td>0.66</td>
<td>0.33</td>
</tr>
<tr>
<td>St Dev</td>
<td>0.40</td>
<td>0.05</td>
<td>0.22</td>
</tr>
</tbody>
</table>

AR coefficient (Cons) 0.22 0.47 0.10

Note: The PSID AR coefficients are estimated after removing household and time fixed effects. All model moments are based on data simulated at the quarterly frequency and then converted to biennial data.

### Table 7
Lagged Expenditure and Spending Propensities in the Model.

<table>
<thead>
<tr>
<th>Dependent variable:</th>
<th>Models</th>
</tr>
</thead>
<tbody>
<tr>
<td>log(consumption)</td>
<td>PSID</td>
</tr>
<tr>
<td></td>
<td>(1)</td>
</tr>
<tr>
<td></td>
<td>(2)</td>
</tr>
<tr>
<td>log (Inc)</td>
<td>0.163</td>
</tr>
<tr>
<td></td>
<td>0.170</td>
</tr>
<tr>
<td>L. High Exp. Episode</td>
<td>0.486</td>
</tr>
<tr>
<td>log(Inc) X L. High Exp Episode</td>
<td>-0.039</td>
</tr>
<tr>
<td>log(Inc) X log(wealth) X log(inc)</td>
<td>NO</td>
</tr>
<tr>
<td>R²</td>
<td>0.65</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Saving Constraint</th>
<th>Bewley</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td></td>
<td>(5)</td>
<td>(6)</td>
</tr>
<tr>
<td>log (Inc)</td>
<td>0.238 -0.126</td>
<td>0.228</td>
</tr>
<tr>
<td>L. High Exp. Episode</td>
<td>0.077</td>
<td>0.030</td>
</tr>
<tr>
<td>log(Inc) X L. High Exp Episode</td>
<td>-0.038</td>
<td></td>
</tr>
<tr>
<td>Control for log(wealth) X log(inc)</td>
<td>NO</td>
<td></td>
</tr>
<tr>
<td>R²</td>
<td>0.68 0.69</td>
<td>0.99</td>
</tr>
</tbody>
</table>

Note: A high expenditure episode is a dummy that equals unity when a household experiences high expenditure but not high income. All regressions control for log(wealth). Standard errors not shown since regression estimates are highly precise. Models are simulated at the quarterly frequency and then converted to biennial data before estimating regressions.

The dependence of spending propensities on lagged expenditure in the saving constraint model is based on the relationship between persistent minimum consumption thresholds.
and MPCs. The theory implies that saving constraints (binding minimum consumption thresholds) are associated with lower spending propensities. Saving constraints are persistent (due to the persistence of the minimum consumption threshold), which implies that lagged saving constraints are also associated with lower spending propensities. High expenditure episodes are a proxy for saving constraints: when households receive a high realization of $c$, their expenditure increases even in the absence of corresponding income increases. Therefore, the theory implies that a high expenditure episode, which is a proxy for a saving constraint, tends to be associated with lower future spending propensities. Below we examine these relationships in more detail using quarterly data generated by the model.

4.1 Analysis Based on Quarterly Data

The advantage of analyzing quarterly data is that we can observe whether households are saving-constrained at any point in time. When aggregating to the two-year frequency, households may move in and out of being constrained over 8 quarters, and hence their 8-quarter consumption will not match their 8-quarter consumption threshold even if they experience episodes of saving constraints and those episodes affect future spending behavior.

Table 8 shows summary statistics and regression coefficients based on the quarterly data produced by the model. Rows (1) and (2) demonstrate that replacing a high expenditure episode with a direct indicator of being saving constrained (specifically, $c_t = c_{t-1}$) leads to a similar pattern in the regressions: lagged saving constraints are associated with lower spending propensities. Approximately 17% of households are saving constrained in any given quarter (row (3)), which implies that a substantial share of households have MPCs near zero. One percent of households experience saving constraints for eight consecutive quarters (column (4)). This persistence helps explain how the effects of savings constraints can be detected even in time-aggregated data.
Table 8
Summary Statistics and Regression Coefficient simulated model (quarterly data)

<table>
<thead>
<tr>
<th></th>
<th>Saving Constraint Model</th>
<th>Bewley Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Coefficient on Interaction between log(income) and saving constrained</td>
<td>-0.34</td>
<td>NA</td>
</tr>
<tr>
<td>(2) Interaction Coefficient as fraction of average Spending Elasticity</td>
<td>-2.43</td>
<td>NA</td>
</tr>
</tbody>
</table>

Share of agents who

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<tbody>
<tr>
<td>(3) Are on saving constraint in a given quarter</td>
<td>0.17</td>
<td>NA</td>
</tr>
<tr>
<td>(4) Are on saving constraint for eight consecutive quarters</td>
<td>0.01</td>
<td>NA</td>
</tr>
<tr>
<td>(5) Have high consumption</td>
<td>0.19</td>
<td>0.18</td>
</tr>
<tr>
<td>(6) Have high consumption but not high income (episode)</td>
<td>0.09</td>
<td>0.04</td>
</tr>
<tr>
<td>(7) Pay the utility cost (consumption&lt; minimum threshold)</td>
<td>0.04</td>
<td>NA</td>
</tr>
</tbody>
</table>

Correspondence between high consumption episode and saving constraint:

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</thead>
<tbody>
<tr>
<td>(8) Incidence of episode if saving constrained</td>
<td>0.42</td>
<td>NA</td>
</tr>
<tr>
<td>(9) Incidence of saving constraint if episode</td>
<td>0.74</td>
<td>NA</td>
</tr>
</tbody>
</table>

Note: In rows (1) and (2), coefficients are based on quarterly regressions with log(consumption) as the dependent variable. Regressions control for log (wealth) and household fixed effects.

Saving constraint episodes do not perfectly correspond to high expenditure episodes. While the incidence of being saving constrained is 17%, the incidence of a high expenditure episode is only 9% (row (6)) because it is possible to be saving constrained even with low levels of consumption (if, for example, income falls but \( c_t = c_{t-1} \) is positive). How well does a high expenditure episode proxy for saving constraints? Approximately three-fourths of households that have a high-expenditure episode but are also saving constrained (row (9)), suggesting that the proxy used in the regressions does indeed capture a large share of households that have MPCs of zero.

5 Consumption Responses to Temporary Income Shocks

Here we examine the implications of saving constraints for the dynamic effects of stimulus measures. We also demonstrate how our model can help explain otherwise puzzling evidence that for many low-income households, anticipated reductions in income are associated with large declines in consumption.

The model with saving constraints features lower average spending propensities than would occur in an environment without minimum consumption thresholds. This implies that stimulus measures, such as one-time transfers, should be less effective in the short-run. Figure 6 simulates the effect of a one-time unanticipated transfer to all households in the saving constraint model and in the standard Bewley model. Impact spending is indeed much
lower (by over half) in the saving constraint model. But despite stimulus being less effective in the short-run, its effects are more persistent. Saving-constrained households eventually move away from the constraint and use additional income to increase their consumption.

![Figure 6](image)

**Figure 6**

Note: The figure shows the effect on aggregate consumption of a one-time wealth transfer of 0.5 to all households. Each line is the log difference between the average consumption path with and without the transfer.

In the saving constraint model, what is the heterogeneous effect of the transfer? Figure 7 shows the average response by saving constraint status, starting at the ergodic distribution. For each group (saving-constrained, unconstrained, and paying-the-utility cost), the consumption response to the transfer is expressed as an elasticity with respect to average group income in the period of impact. Paying-the-utility-cost households have a large initial response, but within a year, consumption returns to the level of the control group (that does not receive transfer). In other words, these households immediately consume the transfer, without substantially changing their medium/long-term prospects. Saving-constrained households, on the other hand, initially save all of the transfer, allowing them to increase consumption over a longer horizon. That is, their consumption response is hump-shaped. Unconstrained households have a moderate initial increase in consumption, which dissipates slowly. By about 20 quarters, the consumption response of saving-constrained households exceeds that of unconstrained ones. This illustrates a key consequence of binding minimum consumption levels: they render households saving-constrained in the sense that they have pent-up demand for saving. Transfers help them save in the short-run and consume over longer horizons.
Figure 8 repeats the exercise of Figure 7 but defines groups by wealth quintile instead of saving constraint status. Echoing the U-shaped MPC by wealth from Figure 5 and Misra and Surico (2014), the consumption response to the transfer is non-monotonic in wealth. While the bottom 20% poorest households have the largest initial response, the 20–40% by wealth have the weakest initial response, as they are more likely to be saving-constrained. The richest 20% of households are unlikely to be saving-constrained and have the second highest initial response of consumption to the transfer. However, since many of the poorest households are effectively hand-to-mouth, the average response of that group quickly dissipates and within a few quarters the poorest households have the lowest response.

![Figure 7](image)

**Figure 7**

Note: The figure shows the average consumption response to a one quarter unanticipated transfer of 0.5 for saving-constrained households (solid, \( |c - \bar{c}| < 0.00001 \)), unconstrained households (dashed, \( c \geq \bar{c} + 0.00001 \)), and households paying the utility cost (dotted, \( c \leq \bar{c} - 0.00001 \)), expressed as an elasticity with respect to initial income. For each group, the line is the log difference between the average response of households with and without the transfer, divided by .5 over average group income, starting at the ergodic distribution.

We also confront the model with the evidence in Ganong and Noel (2019) that anticipated declines in income have substantial effects on consumption, especially for low-income households. We assume that households experience a decline in their labor income of 25 percent in period 5 which remains until period 15, when it returns to normal. In Figure 9, we show the decision rules for consumption for the highest three values of \( c_t \) and the lowest income levels for \( z_t \) and \( x_t \) – the curves are the differences between the consumption function used in period 4 versus period 5. For low enough wealth, consumption falls when labor income declines, despite it being completely anticipated (as of period 1). In contrast, for values of \( c_t \) that are negative (and therefore do not matter), the consumption functions
Figure 8
Note: The figure shows the average consumption response to a one quarter unanticipated transfer of 0.5 by wealth, expressed as an elasticity with respect to initial income. Line thickness corresponds to wealth quintile, so the thinnest line represents the bottom 20% by wealth in the ergodic distribution. For each group, the line is the log difference between the average response of households with and without the transfer, divided by .5 over average group income, starting at the ergodic distribution.

are identical. Thus, we show that even conditional on wealth, consumption will fall in our model with anticipated income declines for poor-enough households.

Figure 9
Note: The figure shows the consumption function in the period prior to an anticipated income tax relative to consumption in the period of the income tax, for low-income, high-g households. A positive value indicates that consumption falls upon impact of the anticipated income tax.
6 Conclusion

A common anecdote referenced by policymakers and politicians is of an American household with limited financial resources that is susceptible to an adverse shock, such as a health expense or a broken automobile, that causes the household to accumulate debt. This debt is burdensome in the sense that additional income is allocated to debt service (net asset accumulation) rather than additional consumption. The poorest of these households are often considered especially vulnerable because they must forego medical care, food, or other basic necessities in the face of adverse shocks.

We highlight the relevance of such households by documenting a number of striking features of the microdata. First, household-level expenditure is nearly as volatile as income. Second, household-level expenditure is relatively uncorrelated with income. Third, a large share of low-wealth households have MPCs near zero. Fourth, we document that lagged high expenditure is associated with low contemporaneous spending propensities. Each category of expenditure contributes to this pattern.

To explain these facts, we develop a theory of saving-constrained households. The theory incorporates time-varying minimum consumption thresholds that, if violated, yield substantial utility costs. Households that experience a high minimum consumption threshold (relative to their wealth) increase consumption and debt. In order to avoid the potential utility cost of violating the minimum consumption threshold in the future (due to insufficient wealth/excess leverage) saving-constrained households buffer themselves by saving rather than spending out of additional income.

An implication of our theory is that a large share of high-debt households are saving-constrained rather than credit-constrained. As a result, income transfers to low-income, high-debt households are less expansionary in the short-term (a few months) than previous models of incomplete markets with heterogeneous agents would predict.

Our theory also helps explain otherwise puzzling consumer behavior documented by recent empirical work. In particular, Ganong and Noel (2019) show that, among low-wealth households receiving unemployment insurance (UI) benefits, spending drops precipitously upon the predictable expiration of UI benefits. Households in our model exhibit similar behavior if their income is low and consumption is at the minimum consumption threshold. In that case, a decline in income (even if anticipated) forces households to consume below the threshold and pay the utility cost.

The prevalence of saving-constrained households is likely to have implications for the value of social insurance and for the propagation of macroeconomic shocks. For example, in a companion paper, Miranda-Pinto et al. (2019), we embed savings constraints in a general equilibrium framework to help explain cross-country differences in the macroeconomic response to fiscal shocks.
References


A Computational Appendix

The recursive problem for the household can be written as

\[ v(a, z, x, \zeta) = \max_{a', c} \left\{ c^{1-\sigma} - \frac{1}{1-\sigma} - \lambda \max \{ \zeta - c, 0 \} + \beta E \left[ v(a', z', x', \zeta') \right] \right\} \]

subject to the budget constraint

\[ c + a' \leq (1 + r + \phi 1(a < 0)) a + w \exp(z) \exp(x) \bar{h}, \]

the borrowing constraint

\[ a' \geq b, \]

and the processes for \( z, x, \) and \( \zeta. \) Assume each of these variables follows an AR(1):

\[ y' = (1 - \rho y) \mu_y + \rho_y y + \sigma_y \epsilon_y \]

with \( \epsilon \) a standard normal, for \( y \in \{ z, x, \zeta \}. \) We then approximate each process as a Markov chain using Rouwenhorst’s method with \((3,3,7)\) states, respectively.

We iterate on the Bellman equation to solve the recursive problem, using Howard’s improvement steps. We approximate \( v \) using Piecewise-Cubic Hermite polynomials in \( a \) (over an irregularly-spaced grid on \([b, \bar{a}]\) that contains \( 0 \)) and solve the maximization using Feasible Sequential Quadratic Programming (FSQP). Since the kink in the objective function slows down SQP methods substantially (they rely on local quadratic approximations which are inaccurate around the kink), we use a two-part procedure – we first ignore the \( \lambda \max \{ \zeta - c, 0 \} \) term and compute the optimal decisions, then if the optimal \( c \) satisfies \( c < \zeta \) we impose \( \zeta \) as an upper bound in FSQP and resolve.

To compute the distribution, we use the method from Young (2010) – we linearly interpolate the decisions onto a dense evenly-spaced grid and use histograms to approximate the distribution of \( a \) given \((z, x, \zeta)\). For some moments, such as the regression coefficients, we use a stochastic simulation constructed with 4500 households tracked for 16 years, sampled every other year (as in the PSID). The initial conditions for this sample are drawn from the stationary distribution, and we assume that the value for \( x \) does not change over the sample.

For impulse responses, we solve the sequential version of the household problem:

\[ v_t(a, z, x, \zeta) = \max_{a', c} \left\{ c^{1-\sigma} - \frac{1}{1-\sigma} - \lambda \max \{ \zeta - c, 0 \} + \beta E \left[ v_{t+1}(a', z', x', \zeta') \right] \right\} \]

for \( t \in \{1, ..., T\} \) and where

\[ v_T(a, z, x, \zeta) = \max_{a', c} \left\{ c^{1-\sigma} - \frac{1}{1-\sigma} - \lambda \max \{ \zeta - c, 0 \} + \beta E \left[ v(a', z', x', \zeta') \right] \right\}. \]
We set $T$ very large so that the wealth dynamics have converged well before the horizon ends.

To estimate the model, we use DiRDFN, which is based on the DiRect (Divided Rectangles) optimizer and includes general constraints (see Di Pillo et al. 2016). The DiRect algorithm takes a feasible space of parameters (a hyperrectangle) and subdivides it iteratively, and then DiRDFN adds derivative-free local searches with active set methods to handle the constraints. The algorithm is globally convergent to the global minimum, but since the bounds matter (even if they end up not binding, they can affect the search process) we check that the solution does not change if the bounds are increased. The parameters we estimate are $(\beta, \mu_c, \rho_c, \sigma_c, \lambda)$. 