Comparative Profitability of Product Disclosure Statements

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In insurance industry, product disclosure statements (PDSs) consist of descriptions of uncertain contingencies by the insurance plans (e.g., “hospital coverage”, “dental coverage”, etc.) and are often very different. In this paper, we model PDSs as information partitions of the state space, which can influence how a consumer perceives the structure of her choice problem and hence her deductible choices. We study a model of an insurance company that aims to promote profit by designing the framing of its PDS. We compare the company’s profits under two PDSs, one of which is coarser than the other. Our main results show that under simple conditions, the PDS consisting of finer partitions of the more expensive states is more profitable.

Keywords: Insurance demand, framing effect, state aggregation, behavioral economics.

JEL codes: D11, D21, D91

1 Introduction

Different ways of framing the same information have an impact on the final consumer decisions (Hershey et al. 1982; Thaler 1980; Tversky and Kahneman 1981), implying firms should pay close attention to how the product information is presented in the first place.

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Indeed, firms can actively use framing to increase their profit (Piccione and Spiegler 2012; Salant and Siegel 2018). For an insurance firm, its PDS describes the terms of an insurance plan and reflects the framing of the related contingencies, and the design of the PDS is often up to the insurer. When the consumers are not indifferent to the framing of the PDS, changes to how contingencies are structured in the insurance plan would affect the firm’s profit. This paper studies the effect on the insurer’s profit that follows from coarsening (refining) its PDS. In our analysis, we propose a decomposition of the total effect into two components: the pure effect from the redistribution of insurance demand inside the newly formed (split) section(s) and the effect from the redistribution of insurance between the sections. This decomposition allows us to identify sufficient conditions for unambiguous prediction of the direction of the total effect, which otherwise would not be possible. Even though firms may often bundle contingencies, our result is clear-cut: Grouping contingencies into expensive sections hurts profit.

In the real world, firms often choose to frame the insurance plans they offer differently. The two examples below illustrate some typical patterns.

First, consider the private health insurance plans in Australia and Canada. Even though both countries have similar public health care systems, the private health insurance plans group contingencies differently. For example, Blue Cross Ontario offers health insurance plans consisting of five categories: “vision,” “dental,” “drugs,” “hospital,” and “massage therapy.” At the same time, Australian insurer NIB offers a coarser version of Blue Cross Ontario’s plan that consists only of two categories: “hospital” and “extras” (vision, dental, physio, pharmacy). In both situations, to buy an insurance plan, the consumer has to visit the company’s website and make her choice inside each category, where the precise details of the plan are specified in fine print.

Second, consider dental insurances. Manulife, a major Canadian insurer, offers a dental insurance policy that groups all relevant contingencies into six categories: “preventive,” “restorative,” “endodontic and periodontics,” “major,” “orthodontia,” and “implants & related.” At the same time, the US insurer Spirit Dental & Vision combines Manulife’s categories “endodontic and periodontics,” “major,” and “implants & related” into its own “major,” and hence offers to the consumers a coarser description of the plan listing only four sections:
In both scenarios, even though we can observe the insurance firms choose different frames of otherwise similar policies, it is unclear which ones are more profitable and why. To answer these questions, we consider a representative consumer who is sophisticated enough to read the fine print, and yet the initial categorization of events still influences her choices. In particular, her deductible choices are generated in two steps—she first evaluates the policy for each category and then aggregates the categorial values into an overall evaluation, displaying aversion to value variations both within each category and across different categories. Hence, this consumer’s choices depend on how the categories are framed (Burkovskaya, Forthcoming). If the insurer understands the nature of the consumer’s reactions to framing, can he improve profit by administering simple changes to the design of the PDS? In this paper, we aim to find out whether the insurer can benefit from (dis)aggregating categories of an existing PDS.

Formally, the consumer’s behavior in this paper is governed by the State Aggregation Subjective Expected Utility (SASEU) model (Burkovskaya, Forthcoming). The SASEU consumer perceives the state space as a collection of events/“small worlds” that is imposed on her by the PDS the insurer offers. While the consumer is fully aware of all the states of the world, she also reacts to the framing. The agent evaluates each insurance plan in a two-stage process by first computing the expected utility of the plan in each section and then calculating the expected utility across the sections while applying some aggregation function to the values of the sections.

The insurance firm is a price-taker that offers plans consisting of a bundle of deductibles for each state of the world. Linear pricing of the deductibles determines the insurance premium. Importantly, the company is considering redesigning its current PDS to improve profit. It does so by applying one-step aggregation (or disaggregation) of the PDS, that is, combining several sections of the current PDS into one (or splitting one section into several). We are interested in whether such a change in PDS increases or decreases the firm’s profit.

To study the effect of a change in PDS on the insurer’s profit, we propose a decomposition method reminiscent of the Slutsky decomposition in classic demand analysis. The

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1Burkovskaya et al. (2020) find experimental evidence that consumers are not indifferent to the framing of contingencies in insurance plans.
The total impact on profit is decomposed into two parts: the aggregation effect and the event-smoothing effect. The aggregation effect accounts for rearranging insurance demand across states in the newly (dis)aggregated section(s), while keeping the income allocated to the newly (dis)aggregated section(s) unchanged. The event-smoothing effect accounts for redistribution of income across all sections of the new PDS.

We analyze the proposed effect in two cases. First, we consider the case when the utility function is constant absolute risk averse (CARA) and the aggregation function is decreasing absolute risk averse (DARA). Second, we allow the utility function to be DARA, and assume the states and sections can be aggregated monotonically in prices.

In both cases, our results suggest that for risk-averse and aggregation-averse consumers, the aggregation effect on insurer’s profit is negative when aggregated states form an expensive event. The concave aggregation function forces the consumers to care more about the overall utility within each section and the balance across different sections. Consequently, to achieve a greater overall utility within a section, a consumer would redistribute her consumption from the more expensive states to the cheaper ones. As a result, such a consumer would redistribute her insurance demand inside the newly aggregated section from the more expensive states to the cheaper states. Hence, the insurer will be losing money. After this aggregation, the distribution of event values within the aggregated section changes from stochastic to deterministic and experiences a second-order stochastic dominance improvement. Due to the DARA assumption, the consumer displays positive prudence in the aggregation function and responds to the aggregation by redistributing consumption from the newly aggregated section to the other sections of the new PDS. Whenever the newly aggregated section is expensive enough to dominate the event-smoothing effect, in the second step, the insurance company would be losing money again. To summarize, if the insurance company is dealing with an aggregation-averse and prudent consumer, aggregating sections of its PDS into expensive ones would result in greater loss. Similarly, disaggregating expensive sections of the PDS will imply all the above-mentioned effects in the opposite direction, and lead to higher profit.

Our main findings can shed light on the insurance company’s contract design. The results imply that, when consumers are aggregation averse and prudent, an insurance company
should avoid aggregating events that are expensive when designing its PDS. Although other small costs could be associated with providing a finer PDS, such as a slightly higher menu cost or higher cognitive cost borne by the consumers, the change we propose is easy to implement—it only requires rewriting the insurance booklet, breaking down the expensive sections. Importantly, it clearly profits the firm. By contrast, the standard techniques that an insurer normally uses to raise profit—such as coming up with a new product, increasing premiums, or investing in obtaining information that would allow the insurer to better discriminate consumers—can be more costly to implement and may even risk the company’s market share. By comparison, the change we propose is a simple remedy for the firm.

We also provide general characterizations that will help the firm quantify the effect on profit from aggregation, even if neither the monotone pricing nor the CARA utility function and the DARA aggregation function assumption is convincing (section 4.3). Again the quantitative results can be useful for guiding insurance contract design for a wider family of preferences.

Finally, the decomposition method we develop for the analysis can be useful for demand analysis for preferences beyond the SASEU case. The method applies to any decision problem where optimal decisions depend on how uncertain states are partitioned and the agent evaluates utility recursively in two stages, (e.g., Li (2020))\(^2\)

### 1.1 Literature review

Our paper builds on the decision theory literature on the framing effect. The phenomenon that the framing of unknown contingencies could affect choices has been observed for a long time (Hershey et al., 1982; Thaler, 1980; Tversky and Kahneman, 1981). A few more recent papers provide axiomatic foundations for this effect. Salant and Rubinstein (2008) take as primitive extended choice functions defined on a product domain of menus and (abstract) frames, where each function assigns a pair of menu and frame to an object. They consider the choice correspondence induced by an extended choice function and characterize conditions under which it is rationalizable. A related paper is Bernheim and Rangel (2009), who provide

\(^2\)Li (2020) considers recursive utilities (under ambiguity) that depend on both the state-contingent outcomes and how uncertainties are gradually resolved in two stages.
more welfare analysis. Alternatively, Ahn and Ergin (2010) model frames as partitions of the state space and consider frame-dependent preferences over acts. Their goal is to characterize a partition-dependent expected utility representation in which the agent’s likelihood judgement of events is described by a non-additive belief function. Similarly, Burkovskaya (Forthcoming) also models frames as partitions. She characterizes an SASEU representation for preferences over acts, in which the agent evaluates an act in two stages: first for each event and then in the aggregate. The current paper assumes that in the insurance market consumers have frame-sensitive preferences that admit the SASEU representation, while the focus is on how an insurer can profit from cleverly designing the frames.

Our paper is closely related to the literature that studies the market interactions between profit-maximizing firms and boundedly-rational consumers. Piccione and Spiegler (2012) analyze a model of two firms engaging in price competition, while the comparability of their prices is limited as a result of different frames chosen by the firms. They identify a condition on the comparability structure of frames (called “weighted regularity”) that guarantees for each firm the existence of a framing strategy that induces the same degree of comparability regardless of its opponent’s frames. In any symmetric equilibrium, they show that this condition is necessary and sufficient for the firms to receive competitive profit. Spiegler (2014) studies a similar model with more general frames induced by arbitrary marketing messages, and provides corresponding characterization of the weighted regularity condition in the general framework. Salant and Siegel (2018) study the design of adverse-selection contract with binary types by a monopolist seller, where the seller can choose a “frame” feature that can increase a product’s attractiveness or highlight a premium product temporarily. They point out that the optimal separating contract often utilizes the framing effect. A common feature among these papers is they all treat frames as some abstract objects that can directly affect consumer tastes (Salant and Rubinstein 2008). In contrast, in our paper, frames are specifically described as partitions of the state space, and a consumer’s frame sensitivity is a consequence of her SASEU preferences. Moreover, the papers above analyze the strategic

\footnote{Li (2020) also studies a two-stage evaluation procedure in the context of ambiguity. The utility representation used here is also reminiscent of the smooth ambiguity model (Klibanoff et al. 2005) or the SOEU (Grant et al. 2009). As a result of the mathematical similarity, our method can also be applied to these preference models.}
interactions among one or two firms and consumers, in which case firms have the market power to influence prices, whereas our problem’s focus is on the framing decision of a small insurance firm who is a price taker in a perfectly competitive financial market. Finally, a paper that also studies a competitive financial market with cognitive limited consumers is Gul et al. (2017). The authors consider a Lucas-tree economy with boundedly-rational consumers who can only choose coarse consumption plans with respect to some partition, and characterize its competitive equilibrium. The main difference with our paper is that the (framing) partition is determined as some optimized outcome by the consumer in Gul et al. (2017), whereas it is designed by the firm in our work.

Broadly speaking, our paper can also be placed in the information design literature (Bergemann and Morris, 2019; Kamenica and Gentzkow, 2011). It is particularly related to several recent papers that analyze the design of information disclosure to non-expected-utility maximizing agents (Lipnowski and Mathevet, 2018; Beauchêne et al., 2019; Lipnowski et al., 2020; Duraj and He, 2020). In this paper, the sender (insurer) aims to design the PDS, which is an information partition, that is beneficial when the receiver (consumer) is not an expected-utility maximizer. Our paper finds a natural family of consumers’ preferences in which the insurer benefits from providing a finer information partition of the more expensive states.

The paper proceeds as follows. Section 2 introduces the model and the decomposition method. Section 3 states the main result under the CARA utility assumption and illustrates the model with numerical examples. Section 4 provides a discussion about optimal PDS, delivers the results under monotone pricing and also DARA utility, and quantifies the aggregation and event-smoothing effects in the general case.

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4Here we present the model as an insurance demand problem in an insurance market that trades state-contingent claims, yet it is straightforward to translate our analysis to characterize the partial equilibrium in a canonical complete market model with frame-sensitive consumers.
2 The model

2.1 Notation

There are $n$ states of the world that might happen at a future date and the state space is $\Omega = \{s_1, s_2, \ldots, s_n\}$. Let state $s_1$ be the default state at which no accident occurs. For each state $s_i$ with $i > 1$, a loss $l_i$ occurs. An insurance company chooses and offers its clients a PDS $\pi = \{s_1, S_2, \ldots, S_N\}$—an information partition of the state space. A PDS consists of events and a default state of no accident $s_1$, where each event is a separate section of the insurance plan. For each section (event) $S_k$ consisting of states $s_{k1}, \ldots, s_{kt}$, the insurance plan describes for every state in this section the insurance coverage and the corresponding deductibles $d_{k1}, \ldots, d_{kt}$. The insurance premium of the plan is $p = \sum_{i=2}^{n} (l_i - d_i) p_i$, where $l_i - d_i$ is the reimbursement claimed by the consumer in state $s_i$, and $p_i$ is the price of a $1$ deductible decrease in state $s_i$.\footnote{Notice $p_i$ is the price of a unit of Arrow security in state $s_i$: Decreasing the deductible by $1$ in some state is equivalent to increasing consumption by $1$ in the same state.}

To simplify notation, we normalize $p_1 = 1 - \sum_{i=2}^{n} p_i$ and denote by $r_i = \frac{p_i}{P(s_i)}$ the probability adjusted relative price of state $s_i$.

2.2 Consumer

A consumer has some fixed initial income $I_0$ and demands insurance coverage when contingencies are described by PDS $\pi$. Our goal is to study the impact on profit, when the frame adopted in the PDS affects the consumer’s insurance demand. Hence, we assume the consumer has SASEU preferences (Burkovskaya, Forthcoming):

$$U(s, d_2, \ldots, d_n; \pi) = P(s_1) \phi(u(s)) + \sum_{i=2}^{N} P(S_i) \phi \left( \sum_{s_j \in S_i} P(s_j | S_i) u(s - d_j) \right),$$

where $u : \mathbb{R} \mapsto \mathbb{R}$ is the von Neumann-Morgenstern (vNM) utility index of money, $\phi : u(\mathbb{R}) \mapsto \mathbb{R}$ is a function that describes the consumer’s attitude toward variation in conditional utilities on events in $\pi$, which we call event risk, $P(s_i)$ is the probability of state $s_i$, $s$ is the consumption in state $s_1$, and $d_i$ is the deductible choice in state $s_i$.

Preferences for categorization or simplification of the state space guide the non-trivial
behavior of the consumer in our paper. The agent is fully aware of the existence of each state; however, she aggregates some of the “similar” states together to create a “small world” relevant for a certain problem. In this paper, the insurer conveniently offers such categorization through the framing of its PDS. Note that even though the consumer groups some states together, she is not restricted to making identical choices in those states.

Formally, an SASEU consumer follows a two-stage procedure in evaluation. She first calculates the expected utility for each section $S_i$ in PDS $\pi$ using a standard vNM utility function, and then computes the overall expected utility across various sections in $\pi$ applying another aggregation function $\phi$.

Moreover, denote by $I = I_0 - \sum_{i=2}^{n} l_i p_i$ the income minus the full insurance premium and the consumer faces the standard linear budget constraint $p_1 s + \sum_{i=2}^{n} p_i (s - d_i) = I$. Let $B(p, I)$ be the set of feasible choices $(s, d_2 \ldots d_n)$ at price $p$ and income $I$. Given PDS $\pi$, the consumer’s insurance demand problem under PDS $\pi$, which we will call (DP-$\pi$), is

$$\max_{(s, d_2 \ldots d_n) \in B(p, I)} U(s, d_2 \ldots d_n; \pi).$$

Note that if $\phi(\cdot)$ is linear, the model reduces to the classical subjective expected utility (SEU) model and the consumer is indifferent to the framing of PDS. Similar to risk-aversion for $u(\cdot)$, the curvature in $\phi(\cdot)$ brings about the attitude toward event-risk. In particular, a concave $\phi(\cdot)$ delivers event-risk aversion—a type of behavior characterized by more “spread-out” consumption choices in each event as a reaction to the aggregation of states. As $\Delta c_i = \Delta s - \Delta d_i$, the effect on deductible choices goes in the opposite direction of that on consumption.

Throughout this paper, we assume both $u(\cdot)$ and $\phi(\cdot)$ are increasing, strictly concave, and three-times differentiable. Moreover, $A_u(x) = -\frac{u''(x)}{u'(x)}$ is the measure of absolute risk aversion, and $A_{\phi}(u) = -\frac{\phi''(u)}{\phi'(u)}$ is the analogous measure of absolute event-risk aversion, whereas $P_u(x) = -\frac{u'''(x)}{u''(x)}$ and $P_{\phi}(u) = -\frac{\phi'''(u)}{\phi''(u)}$ are the measures of absolute prudence and absolute event-risk prudence, respectively.

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Note we allow the deductibles to take on negative values, hence permitting overinsurance in the states with very low prices. This assumption is equivalent to free access to the financial credit markets.
2.2.1 Interpreting the SASEU preferences

Our analysis relies crucially on the assumption that the consumer has SASEU preferences with respect to the frame designed by the insurer. A natural interpretation of such preferences is a boundedly rational consumer with limited attention span, who cannot focus on a large number of contingencies simultaneously. A consequence of her limited attention is the need for categorization of a large state space: The consumer may need to break down the overall state space (“large world”) into smaller events (“small worlds”), and first focus on evaluation of consumption in each small world separately before aggregating these small-world outcomes (Savage, 1954; Burkovskaya Forthcoming). Another closely related implication of limited attention span in face of complex problems is narrow bracketing; that is, when presented with multiple choice problems, people often break them down and only solve one problem at a time (Abeler and Marklein 2017; Stracke et al. 2017; Tversky and Kahneman 1981).

Here we assume that the consumer adopts the event categorization suggested by the insurance company’s PDS, which is supported by salience effect and availability bias documented in behavioral economics (Thaler and Sunstein 2009; Tversky and Kahneman 1974). The assumption is realistic because in the insurance market a consumer typically lacks experience in buying an insurance policy, which is not a familiar everyday task that repeats very often. In addition, the consumer also have limited attention, and hence she may take the partition suggested in the insurer’s PDS as a natural hint of the right mental categorization of the state space.

In practice, a typical insurance PDS often groups similar states into the same category and separates unrelated states. Our consumer, who allocates her attention with respect to this PDS, may be guided by this categorization. Hence, she will focus on the event that consists of similar states relevant for a common issue, (for example, different types of cancer are grouped into the same category “cancer” and evaluated jointly), while temporarily ignoring all the other states that are irrelevant. However, when facing states with little in common, (for example, the state “lung cancer”, which belongs to the category “cancer”, and the state “depression”, which belongs to the category “mental illness”, have less in common and will be perceived separately), the consumer is likely to lose focus and only wish to evaluate one
event at a time. This assumption is supported by various psychological findings where a consumer will find it more difficult to contemplate states from unrelated events at once: for instance, the performance of an agent with limited focus deteriorates from multi-tasking in general (Buser and Peter, 2012) and dealing with dissimilar tasks in particular (Speier et al., 1999), and improves when dealing with tasks that are closely related (Lansman et al., 1983).

In the SASEU model, the need for categorization mentioned above is formally modeled through the lack of event separability in the consumer’s evaluation of consumptions in two unrelated states that belong to different categories/events. However, if the consumer faces all relevant states in an event at once, being fully focused, then this event would be separable from other states. Therefore, the most substantive behavioral postulate that characterizes the SASEU preferences is a restricted version of the celebrated sure-thing principle (Savage, 1954), which imposes separability of the consumer preferences with respect to some (finest) partition. This separability restriction generates the partition used in the two-stage evaluation procedure in the SASEU functional. In each stage, the evaluation functional takes the standard expected-utility form, which is characterized by standard expected utility axioms.

Finally, we assume the two key indices in the SASEU functional, \( u \) and \( \phi \), are both strictly concave. The concavity of \( u \) follows from standard risk aversion assumption. At the same time, the concavity of \( \phi \) assumes event-risk aversion, a behavioral notion similar to risk aversion but defined with respect to the distributions of the event values under a partition. Formally, for any fixed partition (with more than a single event), a consumer is (strictly) event-risk-averse if and only if she always (strictly) prefers a mean-preserving contraction of the distribution of the event values for this partition. A behavioral implication for consumption is that after a one-step aggregation, an event-risk-averse consumer chooses a more “spread out” consumption profile inside the aggregated event. The intuition is that after the aggregation the consumer becomes effectively less event-risk averse within the aggregated event, to which she reacts by redistributing her consumption from more expensive states to cheaper ones in the event.

To illustrate different attitudes toward event-risk, consider the following numerical example. Suppose there are three states of the world, \( \Omega = \{s_1, s_2, s_3\} \), with probabilities \( P(s_1) = 0.4, P(s_2) = 0.2, \) and \( P(s_3) = 0.4 \). The consumer can buy an insurance plan con-
sisting of two deductibles $(d_2, d_3)$ corresponding to risky states $s_2$ and $s_3$, whereas state $s_1$ is the default state with no losses. In addition, the Arrow prices of the risky states are $p_2 = 0.3$ and $p_3 = 0.4$. The consumer has income $I = 100$, and her utility is $u(x) = \ln x$. Consider two types of aggregation function: (1) $\phi(u) = u$ is linear; and (2) $\phi(u) = 1 - e^{-u}$ is concave. Table 1 compares the deductible choices for the aggregated PDS $\pi = \{s_1, \{s_2, s_3\}\}$ and the finest PDS $\Omega = \{\{s_1\}, \{s_2\}, \{s_3\}\}$ for the two cases of event-risk attitudes. Observe that the event-risk-averse consumer chooses more “spread-out” deductibles (or consumptions) in the event $\{s_2, s_3\}$ when facing the aggregated frame $\pi$. We hereafter focus on behaviors of the event-risk-averse consumers.

<table>
<thead>
<tr>
<th>$\phi$</th>
<th>$d_2^\Omega$</th>
<th>$d_3^\Omega$</th>
<th>$d_2^\pi$</th>
<th>$d_3^\pi$</th>
</tr>
</thead>
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<td>33.3</td>
<td>66.7</td>
<td>33.3</td>
</tr>
<tr>
<td>concave</td>
<td>34.1</td>
<td>15.6</td>
<td>44.7</td>
<td>8.6</td>
</tr>
</tbody>
</table>

Table 1: Deductible choices for different event-risk attitudes.

2.3 Insurance company

We assume the insurance company is risk neutral and takes asset prices $p_i$ as given. The company aims to maximize expected profit of the following form:

$$profit(\pi) = premium(\pi) - \sum_{i=2}^{n} P(s_i)(l_i - d_i^\pi) = \alpha - \sum_{i=2}^{n} d_i^\pi p_i - \sum_{i=2}^{n} P(s_i)(l_i - d_i^\pi)$$

$$= const - \sum_{i=2}^{n} d_i^\pi (p_i - P(s_i)) = const - L^\pi,$$

where $\alpha$ is a fixed premium for full coverage, $const = \alpha - \sum_{i=2}^{n} l_i P(s_i)$ is a constant part of the profit that does not depend on consumer choices, and $L^\pi = \sum_{i=2}^{n} d_i^\pi (p_i - P(s_i))$ is the expected loss function of the insurer under PDS $\pi$. In the analysis that follows, we work with the loss $L^\pi$, implying its direct relationship with the profit.

Note that generally the insurance company would choose $\alpha$ to maximize profit, or it might come from the market if the company is a price-taker. In the latter case, $\alpha = \sum_{i=2}^{n} l_i p_i$. Nevertheless, the choice of $\alpha$ is irrelevant to the problem of the choice of PDS framing in this model.
2.4 One-step aggregation of PDS

Our goal is to analyze the changes in insurance demand and insurer’s loss resulting from aggregating some events described in a PDS into a coarser event.\footnote{Characterizing the optimal PDS depends on many details and is ambiguous overall. Instead, in this paper, we concentrate on a relatively “small” change in the structure of the current PDS—the case that produces clear-cut predictions. Even though we are not able to provide a complete characterization about the general structure of the optimal PDS, we show some of its necessary features. See section 4.1 for further discussion.}

Denote the original PDS by $\pi = \{s_1, S_2, \ldots, S_N\}$ and the aggregated PDS by $\rho = \{s_1, S_2, \ldots, S_{k-1}, B, S_{t+1}, \ldots S_N\}$, where event $B$ is the union of $S_k, \ldots S_t$ ($k < t$). We assume the default state $s_1$ cannot be aggregated with any other states, because $s_1$ represents the distinctive case when no loss occurs. Throughout the paper, we assume the insurer observes the insurance demand under the current PDS $\pi$, state prices, probabilities, and income of its clients. For the quantitative analysis in section 4.3, we also suppose the insurance company knows or is able to estimate the vNM utility function $u(\cdot)$ and the aggregation function $\phi(\cdot)$. Since we treat the state prices and probabilities as fixed, we will ignore how consumer choices are dependent on them in our notation.

In the beginning, the consumer with the PDS $\pi$ chooses $(s, d_2, \ldots, d_n)$ to solve the insurance demand problem (DP-$\pi$). For simplicity, we denote the solution to this problem as a consumption bundle $c = (s, c_2, \ldots, c_n)$, the optimal deductibles as $d_i = s - c_i$ (for $i = 2, \ldots, n$), and the corresponding loss as $L = \sum_{i=2}^{n} d_i(p_i - P(s_i))$. The following first-order conditions (FOCs) characterize solutions to this $\pi$-optimization problem:

$$\phi'(V_{S_k}(s))u'(s - d_i) = \lambda_\pi r_i \text{ for all } S_k \in \pi, s_i \in S_k,$$

where $\lambda_\pi \geq 0$ is the Lagrange multiplier for the budget constraint $p_1s + \sum_{i=2}^{n} p_i(s - d_i) = I$ in the $\pi$-optimization problem; $V_{S_k}(s) = \sum_{s_i \in S_k} P(s_i|S_k)u(s - d_i)$ is the expected utility conditional on event $S_k$, which we call the value of event $S_k$.

By dividing the FOCs for any two events $S_k, S_l$, we have

$$\frac{r_i}{r_j} = \frac{\phi'(V_{S_k}(s))}{\phi'(V_{S_l}(s))} \frac{u'(s - d_i)}{u'(s - d_j)} \text{ for all } s_i \in S_k, s_j \in S_l.$$
After aggregation, events \( \{S_k, \ldots S_t\} \) are lumped into a single event \( B \). The consumer facing PDS \( \rho = \{s_1, S_2, \ldots, S_{k-1}, B, S_{t+1}, \ldots S_N\} \) solves the problem (DP-\( \rho \)):

\[
\max_{(s,d_2 \ldots d_n) \in B(\rho, I)} U(s, d_2 \ldots d_n; \rho) = P(s_1)\phi(u(s)) + \sum_{s_i \in \rho \setminus \{s_1\}} P(S_i)\phi\left(\sum_{s_j \in S_i} P(s_j|S_i)u(s - d_j)\right).
\]

We denote the optimal solution to (DP-\( \rho \)) as a consumption bundle \( \tilde{c} = (\tilde{s}, \tilde{c}_2, \ldots, \tilde{c}_n) \) and optimal deductibles as \( \tilde{d}_i = \tilde{s} - \tilde{c}_i \). The corresponding loss is denoted \( \tilde{L} = \sum_{i=2}^{n} \tilde{d}_i(p_i - P(s_i)) \).

The FOCs for the \( \rho \)-optimization problem are

\[
\phi'(V_{S_k}(\tilde{s}))u'(\tilde{c}_i) = \lambda_{\rho} r_i \text{ for all } S_k \in \rho \text{ and } s_i \in S_k, \tag{2}
\]

where \( \lambda_{\rho} \geq 0 \) is the Lagrange multiplier for the same constraint \( p_1 s + \sum_{i=2}^{n} p_i (s - d_i) = I \) in the \( \rho \)-optimization problem.

Our main interest in this paper is the changes in insurance demand \( \Delta d_i = \tilde{d}_i - d_i \) and the expected loss of the insurer \( \Delta L = \tilde{L} - L \), following an aggregation of PDS from \( \pi \) to \( \rho \). Note our analysis applies directly to the impact following a disaggregation of PDS from \( \rho \) to \( \pi \).

To begin with, we consider a knife-edge special case—actuarially fair pricing—when framing/PDS has no effect on insurance demand and hence insurer’s profit.

We say a state \( s_i \) has actuarially fair pricing if \( p_i = P(s_i) \). Say the consumer demands full coverage if \( c_i = c_j \) for any \( s_i, s_j \in \Omega \). The next lemma states the observation that when every state has actuarially fair pricing, then demanding full coverage is optimal for any risk-averse and event-risk-averse SASEU consumer.

**Lemma 1.** Suppose all the states have actuarially fair pricing. For all SASEU consumers (with strictly concave and differentiable \( u \) and \( \phi \)) and all PDS \( \pi \in \Pi \), the optimal insurance demand leads to the same demand for full coverage.

**Proof.** Take any PDS \( \pi \), the FOCs for the consumer are

\[
\phi'(V_{S_k}(s))u'(c_i) = \lambda_{\pi} r_i, \quad \forall S_k \in \pi, s_i \in S_k.
\]
Note that \( r_i = 1 \) if and only if state \( i \) is priced actuarially fairly. Hence, consumption is constant \( c_i = c_j \) for any two states \( s_i, s_j \) in the same event \( S_k \in \pi \). Then, for any event \( S_k \in \pi \), let \( c_{S_k} \) denote the constant consumption level within event \( S_k \). By definition, \( V_{S_k}(s) = u(c_{S_k}) \). For two events \( S_k, S_l \in \pi \), the FOCs imply
\[
\phi'(u(c_{S_k}))u'(c_{S_k}) = \phi'(u(c_{S_l}))u'(c_{S_l}).
\]

Since \( \phi \) and \( u \) are strictly concave, we have \( c_{S_k} = c_{S_l} \).

The lemma above says that under actuarially fair pricing, designing PDS has no effect on insurance demand. Nevertheless, we think it is unlikely to happen if the number of states is large, which is often the case in the insurance market. From now on, we will analyze the case when not all states have actuarially fair pricing, and varying frames/PDSs can affect profit.

### 2.5 Decomposition of the effect

We call the effect of a change from \( \pi \) to \( \rho \) on consumption and deductibles a total one-step aggregation effect. We decompose this total effect into two parts—an aggregation effect and an event-smoothing effect. The aggregation effect accounts for the agent’s desire to rearrange consumption according to the newly aggregated event. The event-smoothing effect refers to the agent’s desire to balance the overall values of different sections of the PDS.

The decomposition we use is close in spirit to the classic Slutsky decomposition of the effect on consumption from a change in price. Our aggregation effect is in line with the classic substitution effect, which requires consumption to adjust according to the new level of the marginal rate of substitution dictated by the new price ratio. The aggregation effect requires the consumption to adjust according to the new level of marginal rate of substitution inside the aggregated event dictated by the change in aggregation. In the same fashion, our event-smoothing effect is in line with the classic income effect, which requires the adjustment of income to satisfy the budget constraint. Similarly, the event-smoothing effect requires readjustment of a fixed amount of income across different events. Nevertheless, it differs from the income effect in that the changes in income on the newly aggregated event and
those on the unaffected events are in opposite directions.

To decompose the total effect on consumption in each state $s_i$, $\Delta c_i = \tilde{c}_i - c_i$, we define an intermediate insurance bundle $(s^*, d^*_2, \ldots, d^*_n)$ that is reminiscent of the Hicksian demand, which solves the dual expenditure-minimization problem at a fixed indirect utility level and new prices. In a similar spirit, our intermediate consumption solves the utility-maximization problem with the same fixed expenditure on the aggregated event $B$ and the newly aggregated conditional preferences on $B$. By construction, this change applies only to the states in $B$, and hence for any $s_i \notin B$, $d^*_i = d_i$, $s^* = s$. Formally, for any $s_i \in B$, $d^*$ is the solution to the following intermediate problem, denoted (DP-B):

\[
\max_{\{d^*_i\}_{s_i \in B}} P(B) \phi \left( \sum_{s_i \in B} P(s_i|B)u(s^* - d^*_i) \right) \\
\text{s.t. } \sum_{s_i \in B} p_i(s - d^*_i) = \sum_{s_i \in B} p_i(s - d_i).
\]

We can also define an intermediate consumption bundle $c^*_i = s - d^*_i$ and let $I_B = \sum_{s_i \in B} p_i(s - d_i)$.

The FOCs for the solution of the intermediate bundle $(s^*, d^*_2, \ldots, d^*_n)$ are

\[
\phi'(V_B(s^*))u'(s - d^*_i) = \lambda_B r_i,
\]

where $\lambda_B$ is the Lagrange multiplier for the constraint $\sum_{s_i \in B} p_i(s - d^*_i) = I_B$ in the intermediate optimization problem at event $B$.

Our decomposition first considers the change from $c$ to $c^*$, $\Delta c^* = c^* - c$, which is the pure aggregation effect because consumption in all the states unaffected by the one-step aggregation is kept fixed. We then examine the change from $c^*$ to $\tilde{c}$, $\Delta \tilde{c} = \tilde{c} - c^*$, which is the event-smoothing effect because it requires only redistribution of available income across different events in the PDS. We can define the corresponding intermediate changes in the expected loss, $\Delta L^* = \sum_{i=2}^n \Delta d_i^*(p_i - P(s_i))$ and $\Delta \tilde{L} = \sum_{i=2}^n \Delta \tilde{d}_i(p_i - P(s_i))$, the sum of which is the total change in loss; that is, $\Delta L = \Delta \tilde{L} + \Delta L^*$.  

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3 Comparative statics of one-step aggregation

In this section, we consider the case when the analyst does not observe consumer preferences directly; however, she is willing to make certain assumptions about the utility and aggregation functions. In addition, the analyst has access to insurance demand and price data under the current PDS $\pi$. We are interested in analyzing the impact of one-step aggregation of PDS on profit and deductible choices under the following assumptions on $u(\cdot)$ and $\phi(\cdot)$.

**Assumption 1.** $u(\cdot)$ is CARA.

**Assumption 2.** $\phi(\cdot)$ is (weakly) DARA.

Assumption 1 implies $u(c) = \frac{1}{\gamma} (1 - e^{-\gamma c})$, where $\gamma > 0$ is the coefficient of absolute risk aversion. Assumption 2 guarantees non-decreasing absolute aggregation aversion. In section 4, we discuss the comparative statics when these assumptions are relaxed.

For each event $S_k \in \pi$, denote by $p_{S_k} = \sum_{s_i \in S_k} p_i$ the price of a security that delivers $1$ in every state in event $S_k$, and by $r_{S_k} = \frac{p_{S_k}}{P(S_k)}$ the probability-adjusted price of $1$ consumption in event $S_k$.

Our main result states that under the two assumptions, aggregating states into more expensive events (relative to the default state $s_1$) hurts profit.

**Theorem 1.** Suppose $u(\cdot)$ and $\phi(\cdot)$ satisfy Assumptions 1 and 2. If $r_B \geq r_1 \geq 1$, aggregating the PDS from $\pi$ to $\rho$ strictly increases the expected loss. Moreover, both the aggregation effect and the event-smoothing effect lead to a strict increase in expected loss.

Under simple assumptions on preferences, Theorem 1 offers a straightforward method for the insurance firm to raise profit. To do so, the insurer only needs to slightly tweak its PDS by separating events that are more expensive than the default state $s_1$. Standard methods to promote profit, such as higher premiums, more obfuscated contract terms, or more intricate price-discrimination schemes, can cost the firm heavily in market share or administrative expenses. Unlike these methods, the change prosed here is easy to implement. The firm only needs to redesign the presentation of its policy booklet or website, making sure the
expensive states are listed separately. For example, the category “heart diseases” would normally include a number of very expensive medical conditions such as stroke and heart attack. However, our result suggests the insurer would benefit if these conditions were listed separately.

The following example illustrates the findings in our Theorem 1.

**Example 1.** Consider a consumer who has wealth \( I = \$15 \) and would like to purchase an insurance plan. There are four states of the world \( \Omega = \{s_1, s_2, s_3, s_4\} \), which may occur with probabilities \( P(s_1), P(s_2), P(s_3), \) and \( P(s_4) \).

The consumer’s preferences are represented by the SASEU discussed above, with an vNM utility index \( u(\cdot) \) and an event-aggregation index \( \phi(\cdot) \) that captures sensitivity to framing. To illustrate, we consider a CARA \( u \) and a logarithmic \( \phi \) functions as follows:

\[
u(x) = \frac{1}{\gamma} (1 - e^{-\gamma x}), \quad \phi(u) = \ln(u).
\]

Consider the consumer first chooses insurance and faces the finest PDS \( \pi = \{\{s_1\}, \{s_2\}, \{s_3\}, \{s_4\}\} \). Then, suppose the consumer is provided with a coarser PDS \( \rho = \{\{s_1\}, B, \{s_4\}\} \), where event \( B = \{s_2, s_3\} \) is the aggregate of states \( s_2 \) and \( s_3 \).

We set \( \gamma = 0.3, I = 15, \alpha = 5, P(s_1) = 0.2, P(s_2) = 0.4, P(s_3) = 0.1, \) and \( P(s_4) = 0.3 \). In addition, we consider prices \( p_1 = 0.2, p_2 = 0.5, p_3 = 0.2, \) and \( p_4 = 0.1 \). Table 2 shows the choices of insurance plans and the decomposition of the difference in loss for frames \( \pi \) and \( \rho \).

<table>
<thead>
<tr>
<th>( \pi )</th>
<th>( \Delta L^* )</th>
<th>( \Delta L )</th>
<th>( \Delta L^\prime )</th>
</tr>
</thead>
<tbody>
<tr>
<td>intermediate ( \pi )</td>
<td>0.7076</td>
<td>2.1678</td>
<td>-3.5634</td>
</tr>
<tr>
<td>( \rho )</td>
<td>0.0046</td>
<td>0.0018</td>
<td>0.0064</td>
</tr>
</tbody>
</table>

Table 2: Insurance choices and decomposition changes for different frames.

Observe that in Example 1, the insurer achieves a higher profit under the finer PDS that separates states \( s_2 \) and \( s_3 \) than that under the coarser PDS \( \rho \). Moreover, in the one-step

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9Note that even after normalizing the price by the probability, the price is likely to be very high for these conditions.
aggregation, both aggregation and event-smoothing effects reduce profit. Note also that 
\[ r_1^1 = 1 < 1.4 = r_B^1. \]

The next example illustrates the role of the price condition \( r_1 \leq r_B. \)

**Example 2.** Suppose all else is the same as Example 1 but the prices are \( p_1^2 = 0.2, \) \( p_2^2 = 0.2, \) \( p_3^2 = 0.2, \) and \( p_4^2 = 0.4 \) instead. So Assumptions 1 and 2 still hold, but \( r_1^2 = 1 > 0.8 = r_B^2. \) Then, the choices of insurance plans and the decomposition of the difference in loss for frames \( \pi \) and \( \rho \) become as shown in Table 3.

<table>
<thead>
<tr>
<th></th>
<th>( d_2 )</th>
<th>( d_3 )</th>
<th>( d_4 )</th>
<th>( s )</th>
<th>( \Delta L^* )</th>
<th>( \Delta L )</th>
<th>( \Delta \bar{L} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi^2 )</td>
<td>-2.2349</td>
<td>2.1643</td>
<td>0.9095</td>
<td>10.3497</td>
<td>0.0333</td>
<td>-0.0065</td>
<td>0.0268</td>
</tr>
<tr>
<td>intermediate( ^2 )</td>
<td>-2.3458</td>
<td>2.2752</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \rho^2 )</td>
<td>-2.2807</td>
<td>2.3403</td>
<td>0.9099</td>
<td>10.3759</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3: Insurance choices and decomposition changes for different frames.

Observe that in Example 2, the aggregation effect is still in the same direction as before, but relaxing the price condition might change the sign of the event-smoothing effect.

Together, Example 1 and Example 2 satisfy Assumptions 1 and 2. However, Example 1 satisfies condition \( r_B \geq r_1 \geq 1, \) whereas Example 2 does not. This condition allows us to predict the direction of the loss change for both effects in Example 1, and hence to know unambiguously the direction of the total loss change. At the same time, we are still able to establish the direction of the aggregation effect in Example 2, but we cannot say the same about the event-smoothing effect. Thus, we cannot predict the direction of the loss change in Example 2.

In what follows, we prove Theorem 1 in three steps. First, we show Assumptions 1 and 2 imply monotonic ordering of events with respect to event prices, which allows us to rank events without additional knowledge about the consumer’s utility and aggregation functions. Second, we analyze the aggregation effect and show the consumer redistributes insurance from the more expensive sub-events to the cheaper ones inside the newly aggregated event \( B, \) which reduces the firm’s profit. Finally, we analyze the event-smoothing effect. We verify that the consumer redistributes insurance from the newly aggregated event \( B \) to all events.
unaffected by the aggregation. If $B$ is also expensive enough, this effect further reduces profit.

### 3.1 Monotonicity

The first and most important implication of Assumptions 1 and 2 is that the conditional expected utility of an event is negatively related to its probability-adjusted price, which is stated in the following lemma.

**Lemma 2** (Monotonicity). Under Assumptions 1 and 2, for any two events $S_i, S_j \in \pi$, $V_{S_i}(s) > V_{S_j}(s)$ if and only if $\frac{p_{S_i}}{P(S_i)} < \frac{p_{S_j}}{P(S_j)}$.

The monotonicity property indicated in Lemma 2 allows us to reorder the events according to their prices in the following way. For each event $S_k \in \pi$, $r_{S_k} = \frac{p_{S_k}}{P(S_k)}$ is the probability-adjusted price of $1$ consumption in event $S_k$. Let the events in $\pi$ be ranked from the cheapest to the most expensive; that is, $r_{S_2} < \cdots < r_{S_N}$. By Lemma 2, we have $V_{S_2}(s) > \cdots > V_{S_N}(s)$. Upon aggregation to PDS $\rho$, the price of the aggregated event $B = \{S_k, \ldots, S_t\}$ can be calculated and ranked with events in $\rho \setminus B$. Furthermore, the events inside $B = \{S_k, \ldots, S_t\}$ can be ranked similarly. An attraction of Lemma 2 is that the order of the events can be computed based on observable variables only (i.e., prices and probabilities).

**Example 3.** According to Lemma 2, we can rank events under each frame in the illustrating examples that we give earlier. For the prices in Example 1, we have the following ranking:

\[
\pi^1: \ r_1 = 1; \ r_2 = 1.25; \ r_3 = 2; \ r_4 = 0.33 \Rightarrow V_{s_3}(s) < V_{s_2}(s) < V_{s_1}(s) < V_{s_4}(s)
\]

\[
\rho^1: \ r_1 = 1; \ r_B = 1.4; \ r_4 = 0.33 \Rightarrow V_B(s) < V_{s_1}(s) < V_{s_4}(s).
\]

At the same time, for the prices in Example 2, we have

\[
\pi^2: \ r_1 = 1; \ r_2 = 0.5; \ r_3 = 2; \ r_4 = 1.33 \Rightarrow V_{s_3}(s) < V_{s_4}(s) < V_{s_1}(s) < V_{s_2}(s)
\]

\[
\rho^2: \ r_1 = 1; \ r_B = 0.8; \ r_4 = 1.33 \Rightarrow V_{s_4}(s) < V_{s_1}(s) < V_B(s).
\]
3.2 Aggregation effect

In this subsection, we consider the aggregation effect—$\Delta c^*$ and $\Delta L^*$—under the assumptions on $u$ and $\phi$. We start with Lemma 3 which establishes the aggregation effect on consumption.

**Lemma 3.** If Assumptions 1 and 2 hold, then for any state $s_j \in S_i$ and event $S_i \in B$, the aggregation effect on consumption is

$$
\Delta c^*_j = \frac{1}{\gamma} \left( 1 - \frac{\phi'(V_{S_i}(s))}{\sum_{S_k \in B} \frac{p_{S_k}}{p_B} \phi'(V_{S_k}(s))} \right).
$$

Note Lemma 3 implies the effect on consumption in event $S_i$ depends on the relationship between the original marginal value in this event, $\phi'(V_{S_i}(s))$, and the relative-price-weighted average of marginal values of sub-events inside $B$. This fact also suggests the aggregation effect on consumption is the same for all states in the same original event, which is the result of CARA $u$. In addition, the aggregation effect on consumption in the cheaper events (with the greatest values and the lowest marginal values) will be positive, pushing the already high consumption and event value up even higher. On the other hand, the aggregation effect on the more expensive events (with the smallest values and the greatest marginal values) will be negative, pushing the already low consumption and the event value further down.

For the aggregation effect on deductibles, because consumption in default state $s_1$ does not change, we have $\Delta d^*_i = -\Delta c^*_i$. Thus, deductibles will increase for the more expensive events, and they will decrease for the cheaper events. Such behavior suggests the desire to redistribute insurance from the more expensive events in $B$ to the cheaper ones.

The next proposition characterizes the aggregation effect on the insurer’s losses.

**Proposition 1.** If Assumptions 1 and 2 hold, then

$$
\Delta L^* = \sum_{s_i \in B} P(s_i) \Delta c^*_i = \frac{P(B)}{\gamma} \left( 1 - \frac{\sum_{S_k \in B} P(S_k | B) \phi'(V_{S_k}(s))}{\sum_{S_k \in B} \frac{p_{S_k}}{p_B} \phi'(V_{S_k}(s))} \right) > 0.
$$

First, note the change in losses corresponds to the change in premium, $(-\sum_{s_i \in B} p_i \Delta d^*_i)$, less the change in the expected reimbursement, $(\sum_{s_i \in B} P(s_i) \Delta d^*_i)$. For the aggregation
effect, the premium is fixed; hence, the change in loss depends only on the redistribution of insurance between the states in $B$, and it equals the change in expected consumption.

Second, the direction of the effect is defined by the relationship between the relative price of event $S_k$ in $B$, $\frac{p_{sk}}{p_B}$, and the conditional probability of event $S_k$ given $B$, $P(S_k|B)$. For the more expensive events, the relative prices $\frac{p_{sk}}{p_B}$ will be greater than the corresponding conditional probabilities $P(S_k|B)$. In this case, the consumer redistributes insurance from the more expensive events toward the cheaper events, while increasing the loss of the insurer.

**Example 4.** The aggregation effect is always positive for CARA $u$ and DARA $\phi$, which is the case for both Example 1 and Example 2 (see Tables 2 and 3 above). However, when Assumptions 1 and 2 do not hold, the aggregation affect might become negative. Consider a case with $u(x) = \ln x$ and $\phi(u) = e^{0.1u}$ and the rest of the parameters from Example 1. Table 4 demonstrates the corresponding solutions to the $\pi$-, $\rho$- and intermediate problems together with the aggregation and event-smoothing effects on losses. In this particular scenario, the aggregation effect is negative.

<table>
<thead>
<tr>
<th></th>
<th>$d_2$</th>
<th>$d_3$</th>
<th>$d_4$</th>
<th>$s$</th>
<th>$\Delta L^*$</th>
<th>$\Delta L$</th>
<th>$\Delta L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi$</td>
<td>2.149</td>
<td>5.256</td>
<td>-23.386</td>
<td>9.787</td>
<td>-0.0117</td>
<td>0.0008</td>
<td>-0.0108</td>
</tr>
<tr>
<td>intermediate</td>
<td>2.227</td>
<td>5.062</td>
<td>-23.389</td>
<td>9.788</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho$</td>
<td>2.229</td>
<td>5.064</td>
<td>-23.389</td>
<td>9.788</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4: Insurance choices and decomposition changes for different frames.

### 3.3 Event-smoothing effect

In this subsection, we turn to the event-smoothing effect that takes care of redistributing the income across events in the eventual PDS $\rho$. We start with establishing an important relationship among the Lagrange multipliers of the three problems we analyze.

**Lemma 4.** If Assumptions 2 and 3 hold, $\lambda_B < \lambda_\rho < \lambda_\pi$, $\Delta \tilde{c}_i < 0$ for all $s_i \in B$ and $\Delta \tilde{c}_j > 0$ for all $s_j \in B^c$.

In the case of CARA $u$ and DARA $\phi$, the value of event $B$ in the intermediate problem always dominates the weighted average value of events in $B$ before the aggregation. Because
the marginal event values have the opposite relationship (because $\phi$ is concave), the above-mentioned effect implies $\lambda_B < \lambda_\pi$, so the shadow value of income in (DP-B) is higher than that in (DP-\pi). Consequently, post-aggregation income should be redistributed from the aggregated event $B$ to other events unaffected by the aggregation to mitigate this gap and equalize the marginal event values again in (DP-\rho). This income redistribution implies $\lambda_\rho < \lambda_\pi$, and consumption in states in $B$ will drop, whereas those in states outside $B$ will go up in the event-smoothing stage. Intuitively, after the aggregation stage, the value of event $B$ goes up more than necessary, so the event-smoothing stage brings it slightly down to balance with the values of the unaffected events. Hence, the consumer redistributes income from event $B$ onto other events.

Furthermore, observe that Assumption 2—$\phi$ is DARA—implies $\phi''' > 0$, which generates an income-redistribution effect. The condition is analogous to the notion of positive absolute prudence and its application in the “precautionary saving” problem in classic risk analysis (Kimball, 1990). In our context, the event-attitude index $\phi$ has positive absolute prudence, which implies a similar “precautionary saving motive” for expenditure redistribution between events $B$ and $B^c$—the consumer will save more for consumption in $B$ if more variations are present in event values $V_{S_k}$ for $S_k \in B$. In particular, under the original PDS $\pi$, she experiences variations in conditional utilities from events $S_k \in B$, and these event values are lumped into a constant utility $V_B(s^*)$ after the event-aggregation stage. Hence, the aggregation effect leads to a lower variation in the event values in $B$ that are aggregated by the $\phi$ function, whereas event values outside $B$ remain unchanged. Therefore, at the event-smoothing stage, when the consumer is redistributing expenditure/“savings” from event $B^c$ to event $B$, she perceives lower variation in the event values in $B$, and hence will “save” less for consumption in $B$. Consequently, together with other standard assumptions, $\phi''' > 0$ implies $\Delta I_B < 0$ (i.e., $\lambda_B < \lambda_\pi$) in the event-smoothing stage.

Similar to the aggregation effect, because $u$ is CARA, the changes in consumption are the same for all the states in an event.

**Lemma 5.** If Assumption 2 holds, then for any event $S_k \in \rho$ and $s_i, s_j \in S_k$, $\Delta \tilde{c}_i = \Delta \tilde{c}_j = \Delta \tilde{c}_{S_k}$. 

\[10\text{In fact, } V_B(s^*) \text{ second-order stochastic dominates } \{V_{S_k}\}_{S_k \in B}, \text{ as } V_B(s^*) > V_B(s).\]
Lemma 6 below suggests changes in consumption outside event $B$ can be monotonically ranked by the corresponding event prices. Hence, the cheaper the event, the greater the change in consumption in the event-smoothing stage.

**Lemma 6.** If Assumptions 1 and 2 hold, then for $S_k, S_l \subseteq B^c$, $V_{S_k} > V_{S_l}$ if and only if $\Delta \tilde{c}_{S_k} > \Delta \tilde{c}_{S_l}$.

Given that $\Delta \tilde{d}_i = \Delta \tilde{s} - \Delta \tilde{c}_i$, Lemmas 5 and 6 suggest the sequence of deductible changes in states unaffected by aggregation, $\{\Delta \tilde{d}_i : s_i \in B^c \setminus s_1\}$, is non-decreasing in $i$. Moreover, the deductible sequence crosses zero from negative to positive values exactly once between events $S_k$ and $S_m$: $r_{s_k} < r_1 < r_{S_m}$.

**Proposition 2.** Under Assumptions 1 and 2, if $r_B \geq r_1 \geq 1$, then $\Delta \tilde{L} > 0$.

The result $\Delta \tilde{L} > 0$ depends closely on the price condition $1 \leq r_1 \leq r_B$. To see this claim, observe that inside event $B$, the deductible changes $\{\Delta \tilde{d}_i : s_i \in B\}$ are always positive. Hence, if the prices of the states in $B$ are not too cheap compared to the price of $s_1$—that is, $r_B \geq r_1$—the full sequence of deductible changes $\{\Delta \tilde{d}_i\}_{i=2}^n$ would turn positive at a state before event $B$. In addition, due to Lemma 2, $\Delta \tilde{L}$—the change in loss from event smoothing—is a weighted average of all deductible changes with increasing weights, which leads to the conclusion. In other words, whenever $B$ is expensive enough, the event-smoothing effect on losses is dominated by the redistribution of the insurance from expensive event $B$ to cheaper events, which results in greater losses for the insurer.

The price condition in Proposition 2 accounts for the difference between Example 1 and Example 2. In Example 1, $r_B = 1.4 > r_1 = 1$ and the deductible changes are: $\Delta \tilde{d}_2 = \Delta \tilde{d}_3 = 0.009$ and $\Delta \tilde{d}_4 = -0.0002$. In this example, the events prices are ranked as follows: $r_4 < r_1 < r_B$. We have already established that $\Delta \tilde{d}_2 = \Delta \tilde{d}_3$ because $u$ is CARA and the changes are positive because both states belong to $B$. The fact that $r_4 < r_1$ implies negative value for $\Delta \tilde{d}_4$. And, finally, because $r_1 < r_B$, the sequence of deductibles turns non-negative at state $s_1$, which is right before $B$ in terms of pricing. This fact guarantees the dominance of event $B$ in the event-smoothing effect.

By contrast, in Example 2, $r_B = 0.8 < r_1 = 1 < r_4 = 1.33$ and the deductible changes are $\Delta \tilde{d}_2 = \Delta \tilde{d}_3 = 0.0651$ and $\Delta \tilde{d}_4 = 0.0004$. Even though all deductibles are positive, the
price condition is violated as \( r_B < 1 \), which implies the deductible change will be multiplied by a negative number when calculating the change in loss. Hence, the event-smoothing effect might take any direction in this case.

Propositions 1 and 2 directly imply Theorem 1: When \( u \) is CARA and \( \phi \) is DARA, \( \Delta L = \Delta L^* + \Delta \bar{L} > 0 \) as long as \( r_B \geq r_1 \geq 1 \). The aggregation effect redistributes the insurance from the expensive sections to the cheaper ones inside \( B \), and the event-smoothing effect redistributes the insurance from section \( B \) to the other sections, implying that whenever \( B \) is expensive enough, both effects have the same direction and increase the total loss. An immediate implication is that disaggregating expensive events would lead to a decrease in losses. Hence, the insurer should keep the expensive sections separate from one another.

4 Discussion

4.1 The optimal PDS

So far, we have focused on the comparative statics analysis from a relatively “small” change in the structure of the current PDS. Although a natural question, the optimal PDS depends on more details, and a complete characterization of it is beyond the scope of this paper. Nevertheless, our comparative statics results can shed light on some of its necessary features.

We still focus on the case with CARA \( u(\cdot) \) and DARA \( \phi(\cdot) \). For any PDS \( \pi \), if some non-singleton event \( B \) is more expensive than state \( s_1 \), that is, \( r_B \geq r_1 \), then by Proposition 1 and Proposition 2, disaggregating event \( B \) will increase the profit. The insurer can repeat this process until no room remains to improve profit. This procedure leads to a monotone PDS that better approximates the optimal monotone PDS than the original PDS \( \pi \).

**Corollary 1.** Suppose \( u(\cdot) \) satisfies CARA and \( \phi(\cdot) \) satisfies DARA. In addition, \( r_1 \geq 1 \). Let \( \pi^* \) be an optimal PDS. Then, the optimal PDS does not contain a non-singleton event \( B \) such that \( r_B \geq r_1 \).

**Proof.** Suppose the optimal PDS \( \pi^* \) has such an event \( B \) and \( r_B \geq r_1 \geq 1 \). In this case, \( \pi^* \) cannot be optimal, because by Proposition 1 and Proposition 2, disaggregating event \( B \)
would increase the profit.

**Example 5.** Consider Example 1 again. Now we are interested in which PDS potentially could be optimal. The first step is to calculate $r_B$ for all non-singleton events $B$:

$$
\begin{align*}
    r_{\{s_2, s_3\}} &= \frac{7}{5}; \\
    r_{\{s_2, s_4\}} &= \frac{6}{7}; \\
    r_{\{s_3, s_4\}} &= \frac{3}{4}; \\
    r_{\{s_2, s_3, s_4\}} &= 1.
\end{align*}
$$

In this problem, $r_1 = 1$; hence, an optimal PDS cannot include events $\{s_2, s_3\}$ and $\{s_2, s_3, s_4\}$. In addition, our results allow us to compare some of the PDSs. For example, the frame $\{s_1, \{s_2, s_3, s_4\}\}$ will be worse than any other frame because no matter which events we are aggregating into $\{s_2, s_3, s_4\}$, $r_{\{s_2, s_3, s_4\}} \geq r_1$. Thus, the insurer should consider only $\{s_1, s_2, \{s_3, s_4\}\}, \{s_1, s_3, \{s_2, s_4\}\}$, and $\{s_1, s_2, s_3, s_4\}$ as reasonable PDSs.

Similarly, for the prices in Example 2, we have

$$
\begin{align*}
    r_{\{s_2, s_3\}} &= \frac{4}{5}; \\
    r_{\{s_2, s_4\}} &= \frac{6}{7}; \\
    r_{\{s_3, s_4\}} &= \frac{3}{2}; \\
    r_{\{s_2, s_3, s_4\}} &= 1.
\end{align*}
$$

In this case, the optimal PDS should not contain events $\{s_3, s_4\}$ and $\{s_2, s_3, s_4\}$.

### 4.2 Aggregation of monotone PDS

In this subsection, we relax the assumption that $u$ is CARA, but focus on the special class of monotone PDSs, which are PDSs that only contain events lumped by states with adjacently ranked prices. Again, our goal is to find conditions on preferences ($u$ and $\phi$ functions) and observable variables (e.g., consumption, prices) under which a one-step aggregation of the PDS will lead to a clear prediction of the direction of the profit change.

**Assumption 3 (Monotonicity).** Suppose the states are ranked according to $r_i$; that is, $r_2 < r_3 < \cdots < r_n$. Then, for any event $A$ either in $\pi$ or $\rho$, there exists $i, k \in \mathbb{N}$ such that $i > 1, k > 0$, and $A = \bigcup_{j=i}^{i+k} s_j$.

Under monotonicity, $S_1 = \{s_1\}$, and all other events are obtained by monotonic aggregation of the states. Without loss of generality, we can always relabel states in any PDS to follow the order of the prices, ranging from the cheapest to the most expensive. Monotonicity requires that only states with adjacently ranked prices are aggregated. This assumption
is reasonable in situations when similarly priced states are lumped together. For instance, consider Manulife’s dental insurance described in the introduction. A typical contract groups all covered services into six categories: “preventive,” “restorative,” “endodontic and periodontic,” “major,” “implant and related services,” and “orthodontia,” with services becoming more expensive as one moves from the first to the sixth category.\[1\]

In the same fashion, a standardized government health insurance contract in Australia consists of two parts: “hospital” and “extras.” The first part includes the more expensive states that require hospitalization and the second part covers the cheaper and less serious states such as “physio” or “dental.”

Similar to section \[3\], we decompose the total effect of one-step aggregation on the consumer choice into the aggregation and event-smoothing effects, using the intermediate bundle \((s^*, d_2^*, \ldots, d_n^*)\). Again, we are interested in the signs of the changes in deductibles \(\Delta d_i = \tilde{d}_i - d_i\) and the change in expected loss \(\Delta L = \tilde{L} - L\).

To begin with, we need to rank events and consumption monotonically (by their indices). As Lemma 7 below shows, Assumption 3 allows us to order the events monotonically when \(u\) and \(\phi\) are concave; however, this assumption is not enough to guarantee that the consumption sequence is also monotonically ordered. Hence, we impose the following assumption, which excludes \(\phi \circ u\) to be a lot more concave than \(u\).

**Assumption 4.** For all \(S_k \in \pi\) and all \(x \in \mathbb{R}\), we have

\[
(A_{\phi \circ u}(x) - A_u(x)) \left( \max_{s_i \in S_k} c_i - \min_{s_j \in S_{k+1}} c_j \right) \leq 1 - \frac{\max_{s_i \in S_k} r_i}{\min_{s_j \in S_{k+1}} r_j}. \tag{12}
\]

The next lemma shows that when the normalized state price \(r_i\) is monotonically increasing in \(i\), under concavity of \(u\) and \(\phi\), the event value \(V_{S_k}\) is decreasing in the event index \(k\). If, in addition, \(\phi \circ u\) is not too concave compared to \(u\), the state consumption \(c_i\) is also decreasing in \(i\).

\[1\] In this anecdote, the expenditure for a service is a proxy for the “expensiveness” of the state when it is needed, which should be proportional to its (unobserved) price of the Arrow security.

\[12\] Let \(c_{S_k} = u^{-1}(V_{S_k})\) be the certainty equivalent to consumption on event \(S_k\). When information about certainty equivalent to an event is also available, Assumption 4 can be replaced by the weaker requirement,

\[
(A_{\phi \circ u}(c_{S_{k+1}}) - A_u(c_{S_{k+1}}))(c_{S_k} - c_{S_{k+1}}) \leq 1 - \frac{\max_{s_i \in S_k} r_i}{\min_{s_j \in S_{k+1}} r_j}.
\]
Lemma 7 (Monotonicity). If Assumption [3] holds, then for any $k < l$, $V_{S_k}(s) > V_{S_l}(s)$. Moreover, if Assumption [4] also holds, then for all $i < j$, $c_i > c_j$.

The first part of Lemma 7 follows from the standard arguments given that $\phi \circ u$ is a concave utility function. However, without Assumption [4] consumption belonging to different events may not be monotonically decreasing. To see this claim, let $s_i$ be the last state in $S_k$ and let $s_j$ be the first state in $S_{k+1}$; the FOC yields

$$\frac{\phi'(V_{S_k}(s))}{\phi'(V_{S_{k+1}}(s))} \cdot \frac{u'(c_i)}{u'(c_j)} = \frac{r_i}{r_j}.$$ 

While $\frac{r_i}{r_j} < 1$, $s_i$ and $s_j$ are adjacent states, and hence the ratio can be close to 1. The marginal rate of substitution (in $\phi$) between events $S_k$ and $S_{k+1}$ is also less than 1, due to decreasing event utility and concavity of $\phi$. If $\phi$ is sufficiently concave or the overall consumption decline in the adjacent events is sufficiently large, the marginal rate of substitution (in $\phi$) between events $S_k$ and $S_{k+1}$ can be so small that the marginal rate of substitution (in $u$) between $c_i$ and $c_j$ must be greater than 1. In this case, consumption at the border of two adjacent events can jump upward, whereas consumption within an event is still decreasing. Assumption [4] rules out this possibility.

4.2.1 Aggregation effect

In this subsection, we consider the aggregation effect—$\Delta c^*$ and $\Delta L^*$—under monotone pricing.

Assumption 5. $u(\cdot)$ is (weakly) DARA.

The assumption relaxes Assumption [1] Under Assumption [3] the price-probability ratio $r_i = \frac{p_i}{P(S_i)}$ is monotonically increasing. Assumption [5] implies the risk-adjustment factor $\frac{1}{A_u(c_i)}$ is non-increasing in $i$, because $c_i$ is decreasing in $i$. Hence, $\sum_{s_i \in S_k} \frac{p_i}{A_u(c_i)}$ first-order stochastically dominates $\sum_{s_i \in S_k} \frac{p_i}{A_u(c_i)}$ in event index $k$.

Together with Assumptions [3] and [4] Assumption [5] also allows us to rank changes in consumption according to the prices of states as well. The following proposition suggests monotone pricing could ensure lumping events generates a negative aggregation effect on the insurer’s profit.
Proposition 3. Suppose Assumptions 3-5 hold and $\phi$ is concave. Then, $\Delta d_i^*$ is non-decreasing in $i$ for all $s_i \in B$, and

$$
\Delta L^* = \sum_{s_i \in B} \frac{P(s_i)}{A_u(c_i)} \left( 1 - \frac{\sum_{s_i \in B} \frac{P(s_i)}{A_u(c_i)}}{\sum_{s_i \in B} \frac{p_i}{A_u(c_i)}} \sum_{S_k \in B} \left( \frac{\sum_{s_i \in S_k} \frac{P(s_i)}{A_u(c_i)}}{\sum_{s_i \in S_k} p_i \frac{A_u(c_i)}} \phi'(V_{S_k}(s)) \right) \phi'(V_{S_k}(s)) \right) > 0.
$$

Similar to the CARA-\textit{u} case, the aggregation effect pushes cheaper consumption up and more expensive consumption further down. The only difference from the CARA \textit{u} case is that consumption on each event is monotonically decreasing and no longer constant. The same fact applies to the changes in consumption as well. Intuitively speaking, given that $s$ and $I_B$ remain unchanged, the consumer redistributes insurance from more expensive states to cheaper ones, keeping insurance premium the same and hence hurting the insurer. Formally, this result comes from the following observation: When $u$ is DARA, $\sum_{s_i \in S_k} \frac{P(s_i)}{A_u(c_i)}$ (the risk-adjusted conditional probability) puts heavier weights on the events with the lower consumption and higher prices than $\sum_{s_i \in S_k} \frac{p_i}{A_u(c_i)}$ (the risk-adjusted relative price).

4.2.2 Event-smoothing effect

Next, we turn to the event-smoothing effect that takes care of redistributing income across events in the final PDS $\rho$.

Observe that the event-smoothing effect is closely related to whether $\frac{\lambda}{\lambda_\rho}$ is smaller or greater than 1. Intuitively, when $\lambda_\pi > \lambda_\rho$, the marginal utility of wealth in the (DP-$\rho$) problem is smaller than that from the (DP-$\pi$) problem for events outside $B$. Hence, the consumer is richer in those events under $\rho$ than under $\pi$. Consequently, the event-smoothing effect should result in an increase in income allocated to states outside $B$ and a decrease in income allocated to the states inside event $B$; that is, $\Delta I_B < 0$ and $\Delta I_B^c > 0$.

Assumption 6. $(1/A_u(x))' \leq 1$ for all $x \in \mathbb{R}_+$.

This assumption states that although the absolute risk tolerance (the reciprocal of $A_u(x)$) is non-decreasing in wealth (by Assumption 5), it cannot increase faster than the speed of the wealth increase. Technically, Assumption 6 is equivalent to either of the following two

\footnote{For more details, see Lemma 12 in the Appendix.}
conditions: (i) \(2A_u(x) \geq P_u(x)\) or (ii) \(g(x) = \frac{1}{u'(x)}\) is convex. In our model, Assumption 6 is necessary for clear predictions of the event-smoothing effect. For states inside the aggregated event \(B\), Assumption 6 ensures the risk-adjusted price for each state \(\frac{r_i}{A_u(x)}\) is non-decreasing in \(i\) and consequently shifts the distribution of event values in \(B\) to the right after aggregation. This stochastic shift implies that the marginal value of event \(B\) decreases, and hence the consumer still wants to redistribute income away from event \(B\) after aggregation, even though \(u\) is no longer CARA. Consequently, inside the aggregated event, consumption changes in the event-smoothing step are always negative (Lemma 16 in the Appendix). For states outside \(B\), which are unaffected by the one-step aggregation, consumptions increase in the event-smoothing stage due to the positive income transfer. In this case, Lemmas 18 and 19 in the Appendix show that Assumption 6 is a technical condition needed to ensure the sequence of consumption changes on \(B^c\) is non-increasing.

The next proposition says Assumptions 2-6 together with a price condition, which requires states in the aggregated event \(B\) must be expensive enough, are sufficient for the event-smoothing effect on loss to be positive. We use \(S_{t+1}\) to denote the event in \(\rho\) right after \(B\) in its statement.

**Proposition 4.** Suppose Assumptions 2-6 hold. If \(1 \leq r_1 < \min_{s_j \in S_{t+1}} r_j\), then \(\{\Delta d_i\}_{s_i \in B^c \setminus s_1}\) is a non-decreasing sequence that crosses 0 and \(\Delta \tilde{L} > 0\).

First, similar to the CARA-\(u\) case, aggregation of the events pushes marginal values down in the (DP-\(\rho\)), implying \(\lambda_{\rho} < \lambda_u\) and redistribution of consumption from \(B\) to the events outside \(B\). Moreover, by the discussion above for the states outside \(B\), the sequence of consumption changes \(\Delta \tilde{c}_i\) is non-increasing in \(i\), implying \(\Delta \tilde{d}_i\) is non-decreasing in \(i\) outside \(B\). Thus, the consumer not only distributes consumption from \(B\) to the events outside \(B\), but also distributes more in the cheaper states; that is, she chooses even lower deductibles.

Assumption 6 has also been used in the classic risk analysis as a necessary and sufficient condition for an EU agent to consume more in the face of a new favorable risky investment opportunity tomorrow (Gollier and Kimball 2018). Consider an EU agent with vNM index \(u\), initial wealth \(x_0\), and who faces a favorable risky investment opportunity with return \(\tilde{x}\) in the second period. Then, an agent with \(u\) satisfying Assumption 6 displays the following behavior: For all distributions of \(\tilde{x}\) and all \(x_0\),

\[E[\tilde{x}u'(x_0 + \tilde{x})] = 0 \Rightarrow E[u'(x_0 + \tilde{x})] \leq E[u'(x_0)].\]
in the cheaper states. The redistribution obviously means consumption in the default state goes up as well. In other words, the consumer redistributes insurance not only from $B$ to outside $B$, but also from the more expensive events outside $B$ to the cheaper ones, implying a reduction in the overall premium paid for the insurance.

Second, when event $B$ is so cheap that the price condition $1 \leq r_1 < \min_{s_j \in S_{i+1}} r_j$ fails, the event-smoothing effect on loss cannot be predicted because there are two opposite effects at play: (1) redistribution of insurance from $B$ to other events; and (2) redistribution of insurance from the more expensive events to the cheaper ones. Alternatively, when event $B$ is expensive enough for the price condition to hold, the consumer redistributes insurance only from the more expensive events to the cheaper ones, implying the increase in loss.

To summarize, we have the analogue of Theorem 1 in the monotone PDS case: When the aggregated event is expensive enough, the total effect of a one-step aggregation on the insurer loss $\Delta L = \Delta L^* + \Delta \bar{L}$ is positive under Assumptions [2][6].

4.3 Quantitative analysis

In this section, we provide general characterizations of the aggregation and event-smoothing effects that rely solely on the concavity and differentiability of $u$ and $\phi$.

For that purpose, first, we define the risk-adjusted prices and probabilities that are building blocks of our analysis.

**Definition 1.** For any event $S_j \in B$ and $S_k \in \rho$, the risk-adjusted relative price and conditional probability at $S_j$ are

$$\alpha(S_j) = \frac{\sum_{s_i \in S_j} p_i A_u(c_i)}{\sum_{s_i \in B} p_i A_u(c_i)}$$

and

$$\beta(S_j) = \frac{\sum_{s_i \in S_j} P(s_i) A_u(c_i)}{\sum_{s_i \in B} P(s_i) A_u(c_i)},$$

whereas the post-aggregation risk-adjusted relative price and conditional probability at $S_k$ are

$$\alpha^*(S_k) = \frac{\sum_{s_i \in S_k} p_i A_u(c_i^*)}{\sum_{s_i \in \Omega} p_i A_u(c_i^*)}$$

and

$$\beta^*(S_k) = \frac{\sum_{s_i \in S_k} P(s_i) A_u(c_i^*)}{\sum_{s_i \in \Omega} P(s_i) A_u(c_i^*)}.$$

**Example 6.** If $u(\cdot)$ is CARA, then $\alpha(S_i) = \frac{\sum_{s_j \in S_i} p_j}{\sum_{s_j \in B} p_j} = \frac{p_S}{p_B}$ is the price of $\$1$ consumption in
event $S_i$ relative to that in event $B$ and $\beta(S_i) = \frac{\sum_{s_j \in S_i} p(s_j)}{\sum_{s_j \in B} p(s_j)}$ is the probability of event $S_i$ conditional on event $B$. Alternatively, if $u(\cdot)$ is constant relative risk averse (CRRA); that is, $u(x) = \ln x$ or $u(x) = x^\rho$, then $\alpha(S_i) = \frac{I_{S_i}}{I_B}$ is the share of event $S_i$’s income in $I_B$ and $\beta(S_i) = \frac{\sum_{s_j \in S_i} p(s_j)c_j}{\sum_{s_j \in B} p(s_j)c_j}$ is the ratio of expected consumption on $S_i$ to expected consumption on $B$.

4.3.1 General characterization of the aggregation effect

We start with the aggregation effects on $\Delta c^*$ and $\Delta L^*$—change from the original consumption $c$ to the intermediate bundle $c^*$ and the corresponding change in the firm’s loss.

Define $\Gamma_{S_i} = 1 - \frac{u'(c^*_j)}{u'(c_j)}$, which is the change in marginal (state) utility in state $s_j \in S_i \in B$. Note the first-order conditions require that such change is constant for all the states in an event. The following lemmas provide the exact formula for the change in marginal (state) utility together with the aggregation effects for consumption and loss in terms of consumer’s utility $u$, aggregation function $\phi$, consumptions at the original frame $\pi$, the risk-adjusted relative prices $\alpha(S_k)$, and the risk-adjusted conditional probabilities $\beta(S_k)$.

Lemma 8. If $u$ and $\phi$ are concave and three-times differentiable, then for any state $s_j \in S_i$ and event $S_i \in B$, the aggregation effect on consumption is

$$\Delta c^*_j = \frac{\Gamma_{S_i}}{\phi'(V_{S_i}(s))}.$$ 

$$\Gamma_{S_i} = 1 - \frac{\phi'(V_{S_i}(s))}{\sum_{S_k \in B} \alpha(S_k) \phi'(V_{S_k}(s))}.$$

Lemma 9. If $u$ and $\phi$ are concave and three-times differentiable, then

$$\Delta L^* = \sum_{s_i \in B} P(s_i) \Delta c^*_i = \left( \sum_{s_i \in B} \frac{P(s_i)}{A_u(c_i)} \right) \left( 1 - \frac{\sum_{S_k \in B} \beta(S_k) \phi'(V_{S_k}(s))}{\sum_{S_k \in B} \alpha(S_k) \phi'(V_{S_k}(s))} \right).$$

The direction of the effect is defined by the relationship between the risk-adjusted relative prices $\alpha(S_k)$ and the risk-adjusted conditional probabilities $\beta(S_k)$: If the events with the lower values have “greater” risk-adjusted relative prices than their risk-adjusted conditional probabilities, the aggregation effect on losses will be positive. The losses would increase
because the consumer would redistribute insurance from the relatively more expensive events with lower values toward relatively cheaper events with higher values.

### 4.3.2 General characterization of the event-smoothing effect

Now we move on to the event-smoothing effects on $\Delta \tilde{c}$ and $\Delta \tilde{L}$—change from the intermediate bundle $c^*$ to the consumption bundle $\tilde{c}$ with the PDS $\rho$ and the corresponding change in loss.

Denote by $E_{S_k} = \sum_{s_i \in S_k} P(s_i|S_k) \frac{u'(c_i^*)}{A_u(c_i^*)}$ the risk-adjusted average marginal (state) utility on event $S_k \in \rho$. By analogy to the aggregation effect, define $\tilde{\Gamma}_{S_i} = 1 - \frac{u'(\tilde{c}_i)}{u'(c_i^*)}$ as the change in marginal (state) utility in state $s_j \in S_i \in \rho$. As before, the first-order conditions require that such change is constant for all the states in an event. Lemma 17 in the Appendix provides a system of equations on $\tilde{\Gamma}_{S_i}$ that allows us to pin down the event-smoothing effect using information (input) on consumer’s utility $u$, aggregation function $\phi$, prices, probabilities, consumption choices at (DP-$\pi$) and (DP-B), as well as the ratio of the Lagrange multipliers $\frac{\lambda_B}{\lambda_\pi}$. Note that for state $s_i \in B$, $c_i^*$ can be pinned down by $u$, $\phi$, and $c_i$ from the aggregation effect (Lemma 8), and thus the ratio $\frac{\lambda_B}{\lambda_\pi}$ can be computed. For states $s_i \notin B$, we have $c_i^* = c_i$. Hence, the last two input variables can be obtained from the aggregation effect, and the other input variables are observable. Therefore, the proposed system of equations can be solved computationally in every specific case; unfortunately, obtaining the explicit expression for the solution is not feasible.

Finally, the event-smoothing effect on the insurer losses is as follows.

**Lemma 10.** Suppose $u$ and $\phi$ are three-times differentiable and concave; then,

$$\Delta \tilde{L} = \sum_{s_i \in \Omega} P(s_i) \Delta \tilde{c}_i = \left( \sum_{s_i \in \Omega} P(s_i) \frac{P(s_i)}{A_u(c_i^*)} \right) \sum_{S_k \in \rho} \beta^*(S_k) \tilde{\Gamma}_{S_k}.$$  

Analogous to the aggregation effect, the event-smoothing effect on loss is equal to the expected change in consumption. Lemma 17 requires $\sum_{S_k \in \rho} \alpha^*(S_k) \tilde{\Gamma}_{S_k} = 0$, which simply says that the total income does not change. On the other hand, the direction of the event-smoothing effect on losses depends on the sign of $\sum_{S_k \in \rho} \beta^*(S_k) \tilde{\Gamma}_{S_k}$. Thus, the direction of the effect again depends on the relationship between the post-aggregation risk-adjusted
relative prices $\alpha^*(S_k)$ and the post-aggregation risk-adjusted conditional probabilities $\beta^*(S_k)$; however, this relationship is less straightforward than that in the aggregation effect. Even so, we are still able to see the direction of the effect is determined by whether event $B$ is relatively more expensive than the rest of the events. For example, in the case of $\lambda_B < \lambda_\pi$, if event $B$ is relatively more expensive than the other events, the event-smoothing effect on losses will be dominated by the changes in the unaffected states, which are relatively cheaper and hence would increase the loss of the insurer. On the other hand, if $B$ is relatively cheaper, then $B$ will dominate the direction of $\Delta\tilde{L}$, and the consumer would redistribute the insurance from “cheap” $B$ to “more expensive” other events, which would reduce the insurer’s loss.

Finally, the total effect on the insurer’s loss is simply the sum of the two effects; that is, $\Delta L = \Delta L^* + \Delta \tilde{L}$. The exact quantity of change would depend on different relative prices and conditional probabilities. Hence, even though we cannot predict clearly the direction of profit change for preferences outside the families considered in section 3 or 4.2, the general results in this subsection can educate a firm on how to make quantitative predictions of consumption and profit changes following a one-step aggregation using numerical methods. The method applies to any one-step aggregation (not necessarily restricted to expensive events) and any concave and smooth utility and aggregation functions, as long as the firm can observe current prices, income, and consumption, and is willing to make functional-form assumptions about the consumer’s preferences.$^{15}$

A Appendix

A.1 Notation

To simplify working with the proofs, we list all the notation used below:

$$\alpha(S_j) = \frac{\sum_{s_i \in S_j} p_i A_u(c_i)}{\sum_{s_i \in B} p_i A_u(c_i)} \quad \text{and} \quad \beta(S_j) = \frac{\sum_{s_i \in S_j} P(s_i) A_u(c_i)}{\sum_{s_i \in B} P(s_i) A_u(c_i)} \quad \text{for any } S_j \in B$$

$^{15}$In applied work (e.g., empirical industrial organization), making functional form assumptions about consumers’ utility with some unknown taste shocks is common. The firm can estimate the key parameters of the utility and aggregation functions with consumption data, and then use the estimated utility and aggregation functions as inputs to our quantitative analysis.
\[ \alpha^*(S_k) = \frac{\sum_{s_i \in S_k} \frac{p_{s_i}}{A_u(c_i^*)}}{\sum_{s_i \in \Omega} \frac{p_{s_i}}{A_u(c_i^*)}} \quad \text{and} \quad \beta^*(S_k) = \frac{\sum_{s_i \in S_k} \frac{P(s_i)}{A_u(c_i^*)}}{\sum_{s_i \in \Omega} \frac{P(s_i)}{A_u(c_i^*)}} \quad \text{for any} \ S_k \in \rho \]

\[ \Gamma_{S_i} = 1 - \frac{u'(c_j)}{u'(c_j^*)} \quad \text{for any} \ S_i \in B \]

\[ \tilde{\Gamma}_{S_i} = 1 - \frac{u'(c_j)}{u'(c_j^*)} \quad \text{for any} \ S_i \in \rho \]

\[ E_{S_k} = \sum_{s_i \in S_k} P(s_i | S_k) \frac{u'(c_i^*)}{A_u(c_i^*)} \quad \text{for any} \ S_k \in \rho. \]

### A.2 Special case

#### A.2.1 Aggregation effect

**Proof of Lemma 2.** The FOC suggests

\[ P(s_i)\phi'(V_{S_k}(s))e^{-\gamma c_i} = \lambda \pi p_i, \]

which implies

\[ P(s_i)\phi'(V_{S_k}(s)) \frac{1}{\gamma} (1 - e^{-\gamma c_i}) = \frac{1}{\gamma} (P(s_i)\phi'(V_{S_k}(s)) - \lambda \pi p_i) \]

\[ P(S_k)\phi'(V_{S_k}(s))P(s_i | S_k)u(c_i) = \frac{1}{\gamma} (P(s_i)\phi'(V_{S_k}(s)) - \lambda \pi p_i). \]

By adding up the above expression for all states \( s_i \in S_k \), we obtain

\[ P(S_k)\phi'(V_{S_k}(s)) \sum_{s_i \in S_k} P(s_i | S_k)u(c_i) = \frac{1}{\gamma} (P(S_k)\phi'(V_{S_k}(s)) - \lambda \pi p_{S_k}) \]

\[ P(S_k)\phi'(V_{S_k}(s))V_{S_k}(s) = \frac{1}{\gamma} (P(S_k)(\phi'(V_{S_k}(s)) - \lambda \pi p_{S_k}) \]

\[ \phi'(V_{S_k}(s))(1 - \gamma V_{S_k}(s)) = \lambda \pi \frac{P_{S_k}}{P(S_k)}. \]
Now note the left-hand side is decreasing in $V_{S_k}(s)$ (as $V_{S_k}(s) \in (0, \frac{1}{\gamma})$ and $\phi(\cdot)$ is strictly concave); hence, the greater the ratio $\frac{p_{S_k}}{P(S_k)}$, the smaller the value $V_{S_k}(s)$.

**Proof of Lemma 8.** The ratio of the FOCs between the $\pi$–aggregation problem and the intermediate bundle suggests that for any $s_j \in S_i$ and $S_i \in B$,

$$\frac{u'(c_j^*)}{u'(c_j)} = \frac{\lambda_B \phi'(V_{S_i}(s))}{\lambda_\pi \phi'(V_B(s^*))}.$$ 

Then, by the Taylor expansion, we get

$$1 - A_u(c_j) \Delta c_j^* = \frac{\lambda_B \phi'(V_{S_i}(s))}{\lambda_\pi \phi'(V_B(s^*))}$$

$$\Rightarrow A_u(c_j) \Delta c_j^* = 1 - \frac{\lambda_B \phi'(V_{S_i}(s))}{\lambda_\pi \phi'(V_B(s^*))} = \Gamma_{S_i};$$

hence, $\Delta c_j^* = \frac{\Gamma_{S_i}}{A_u(c_j)}$. Now denote $\gamma = \frac{\lambda_B}{\lambda_\pi \phi'(V_B(s^*))}$ and also notice $\gamma$ does not depend on specific states or events inside $B$. In addition, $\Gamma_{S_i} = 1 - \gamma \phi'(V_{S_i}(s))$.

Denote $\omega(S_k) = \sum_{s_i \in S_k} \frac{p_{S_k}}{A_u(c_i)}$. Finally, note that income in event $B$ is fixed, so $\sum_{s_j \in B} \Delta c_j^* p_j = 0$, which implies

$$\sum_{s_j \in B} \Delta c_j^* p_j = \sum_{s_j \in B} \frac{p_j}{A_u(c_j)} \Gamma_{S_i} = \sum_{S_k \in B} \Gamma_{S_k} \sum_{s_j \in S_k} \frac{p_j}{A_u(c_j)} = \sum_{S_k \in B} \Gamma_{S_k} \omega(S_k) = \sum_{S_k \in B} (1 - \gamma \phi'(V_{S_k}(s))) \omega(S_k) = \omega(B) - \gamma \sum_{S_k \in B} \omega(S_k) \phi'(V_{S_k}(s)) = 0,$$

implying $\gamma = \frac{1}{\sum_{S_k \in B} \omega(S_k) \phi'(V_{S_k}(s))}$, and all the results follow.

**Proof of Lemma 3.** Note Lemma 3 is a corollary of Lemma 8, where we take into account that $A_u(c_i) = \gamma$ and $\alpha(S_k) = \frac{p_{S_k}}{p_B}$ for the case of CARA $u$.

**Proof of Lemma 9.** Because $\sum_{s_i \in B} p_i \Delta c_i^* = 0$ and by taking into account Lemma 8, we
have
\[
\Delta L^* = \sum_{s_i \in B} (p_i - P(s_i)) (-\Delta c^*_i) = -\sum_{s_i \in B} p_i \Delta c^*_i + \sum_{s_i \in B} P(s_i) \Delta c^*_i
\]
\[
= \sum_{s_i \in B} P(s_i) \Delta c^*_i = \sum_{s_i \in B} P(s_i) \sum_{s_i \in S_j} \frac{P(s_i)}{A_u(c_i)} \sum_{s_i \in S_j} \Gamma S_j \beta(S_j) \sum_{s_i \in S_j} \beta(S_j)
\]
\[
= \sum_{s_i \in B} P(s_i) \sum_{s_i \in S_j} \beta(S_j) \left(1 - \frac{\phi'(V_{S_j}(s))}{\sum_{S_k \in B} \alpha(S_k) \phi'(V_{S_k}(s))}\right)
\]
\[
= \sum_{s_i \in B} P(s_i) \sum_{s_i \in S_j} \left(1 - \sum_{S_k \in B} \beta(S_k) \phi'(V_{S_k}(s)) \frac{\beta(S_k)}{\sum_{S_k \in B} \alpha(S_k) \phi'(V_{S_k}(s))}\right).
\]

\[\square\]

**Lemma 11.** Under Assumption 1, \(\frac{\alpha(S_k)}{\beta(S_k)} < \cdots < \frac{\alpha(S_t)}{\beta(S_t)}\).

**Proof.** Note
\[
\frac{p_{S_2}}{P(S_2)} < \cdots < \frac{p_{S_N}}{P(S_N)}
\]
\[
\frac{p_{S_2} P(B)}{p_B} < \cdots < \frac{p_{S_N} P(B)}{p_B}
\]
implies
\[
\frac{\alpha(S_k)}{\beta(S_k)} < \cdots < \frac{\alpha(S_t)}{\beta(S_t)}.
\]
\[\square\]

**Proof of Proposition 1.** By Lemma 9, the aggregation effect on losses is
\[
\Delta L^* = \frac{P(B)}{\gamma} \left(1 - \sum_{S_k \in B} \beta(S_k) \phi'(V_{S_k}(s)) \frac{\beta(S_k)}{\sum_{S_k \in B} \alpha(S_k) \phi'(V_{S_k}(s))}\right).
\]

Now note that by Lemma 11, we have \(\frac{\alpha(S_k)}{\beta(S_k)} < \cdots < \frac{\alpha(S_t)}{\beta(S_t)}\). Thus, \(\{\alpha(S_k) : S_k \in B\}\) first-order stochastically dominates \(\{\beta(S_k) : S_k \in B\}\) in index \(k\). In addition, \(V_{S_k}(s)\) is decreasing in \(k\) and \(\phi\) is a strictly concave function; hence,
\[
\sum_{S_k \in B} \alpha(S_k) \phi'(V_{S_k}(s)) > \sum_{S_k \in B} \beta(S_k) \phi'(V_{S_k}(s)),
\]
and the result follows. \[\square\]
A.2.2 Event-smoothing effect

Lemma 12

The next lemma shows the direction of income redistribution between the aggregated event $B$ and its complement $B^c$ is pinned down by the comparison of the Lagrange multipliers of the two problems $\lambda_\pi$ and $\lambda_\rho$.

**Lemma 12.** If $u$ and $\phi$ are concave and three-times differentiable, one of the following cases holds:

- $\lambda_\pi > \lambda_\rho \iff \lambda_\pi > \lambda_B \iff \Delta I_B < 0$ and $\Delta I_{B^c} > 0$, then $\Delta \tilde{c}_i < 0$ for all $s_i \in B$ and $\Delta \tilde{c}_j > 0$ for all $s_j \in B^c$;

- $\lambda_\pi = \lambda_\rho \iff \lambda_\pi = \lambda_B \iff \Delta I_B = 0$ and $\Delta I_{B^c} = 0$, then $\Delta \tilde{c}_i = 0$ for all $s_i \in B$ and $\Delta \tilde{c}_j = 0$ for all $s_j \in B^c$;

- $\lambda_\pi < \lambda_\rho \iff \lambda_\pi < \lambda_B \iff \Delta I_B > 0$ and $\Delta I_{B^c} < 0$, then $\Delta \tilde{c}_i > 0$ for all $s_i \in B$ and $\Delta \tilde{c}_j < 0$ for all $s_j \in B^c$.

**Proof.** For all $s_i, s_j \in B$, the marginal rate of substitution satisfies

$$\frac{u'(\tilde{c}_i)}{u'(\tilde{c}_j)} = \frac{r_i}{r_j} = \frac{u'(c_i^*)}{u'(c_j^*).}$$

Hence, $\text{sgn}(\Delta \tilde{c}_i) = \text{sgn}(\Delta \tilde{c}_j)$ for all $s_i, s_j \in B$. By the same argument, for arbitrary $s_i, s_j \in S_k \subseteq B^c$, $\text{sgn}(\Delta \tilde{c}_i) = \text{sgn}(\Delta \tilde{c}_j) = \text{sgn}(\Delta V_{S_k}(\tilde{s}))$.

Then in addition for any other event $S_l \subseteq B^c$ and any state $s_j \in S_l$, we have

$$\frac{\phi'(V_{S_k}(\tilde{s})) u'(\tilde{c}_i)}{\phi'(V_{S_l}(\tilde{s})) u'(\tilde{c}_j)} = \frac{r_i}{r_j} = \frac{\phi'(V_{S_k}(s)) u'(c_i)}{\phi'(V_{S_l}(s)) u'(c_j),} \quad \forall s_i \in S_k, s_j \in S_l.$$

By the same argument, $\text{sgn}(\Delta \tilde{c}_i) = \text{sgn}(\Delta V_{S_k}(\tilde{s}))$ for any $s_i \in S_k$. Hence, we get that for any $S_k, S_l \in B^c$ and $s_i \in S_k, s_j \in S_l$: $\text{sgn}(\Delta \tilde{c}_i) = \text{sgn}(\Delta \tilde{c}_j) = \text{sgn}(\Delta V_{S_k}(\tilde{s})) = \text{sgn}(\Delta V_{S_l}(\tilde{s}))$.

Hence, we obtain that event-smoothing effect has the same direction in the events outside $B$ and $\text{sgn}(\Delta V_{S_k}(\tilde{s})) = \text{sgn}(\Delta I_{B^c})$ for any $S_k \in B^c$. 

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Moreover, note $\Delta I_B + \Delta I_{B^c} = 0$. Thus, for any $s_i \in S_k \in B^c$ and $s_j \in B$, we have

$$\text{sgn}(\Delta \tilde{c}_i) = \text{sgn}(\Delta V_{S_k}(\tilde{s})) = \text{sgn}(\Delta I_{B^c}) = - \text{sgn}(\Delta \tilde{c}_j)$$

$$= - \text{sgn}(\Delta V_B(\tilde{s})) = - \text{sgn}(\Delta I_B).$$

Next, consider the ratio of the FOCs for $\pi-$ and $\rho-$aggregation problems for any $S_k \in B^c$:

$$\frac{\lambda_{\pi}}{\lambda_{\rho}} = \frac{\phi'(V_{S_k}(s))}{\phi'(V_{S_k}(\tilde{s}))} \frac{u'(c_i)}{u'(\tilde{c}_i)}.\quad (3)$$

Due to $\text{sgn}(\Delta \tilde{c}_i) = \text{sgn}(\Delta V_{S_k}(\tilde{s}))$ for all $s_i \in S_k \in B^c$ and $s_j \in B$, we get that

$$\frac{\lambda_{\pi}}{\lambda_{\rho}} > 1 \iff \Delta \tilde{c}_i > 0, \Delta V_{S_k} > 0, \Delta I_{B^c} > 0, \Delta I_B < 0, \Delta \tilde{c}_j < 0.$$

For any $s_i \in S_k \subseteq B$, dividing the FOCs (3) and $\rho-$aggregation problems yields

$$\frac{\lambda_B}{\lambda_{\rho}} = \frac{\phi'(V_B(s^*))}{\phi'(V_B(\tilde{s}))} \frac{u'(c^*_i)}{u'(\tilde{c}_i)}.\quad (3)$$

Hence, when $\Delta I_B < 0$, we also obtain $\frac{\lambda_B}{\lambda_{\rho}} < 1$. The other two cases follow by analogy.

Proof of Lemma 4. Note

$$V^*_B(s^*) \geq \sum_{S_k \in B} P(S_k|B)V_{S_k}(s) \quad (4)$$

because $V^*_B(s^*)$ is the optimal value at the intermediate bundle $c^*$. Also note $P(S_k|B) = \beta(S_k)$; hence,

$$V^*_B(s^*) \geq \sum_{S_k \in B} \beta(S_k)V_{S_k}(s) > \sum_{S_k \in B} \alpha(S_k)V_{S_k}(s).$$

The last expression follows from $\frac{\alpha(S_k)}{\beta(S_k)} < \cdots < \frac{\alpha(S_t)}{\beta(S_t)}$ and $V_{S_k}(s) > \cdots > V_{S_t}(s)$.

Now notice $\phi$ is concave and $\phi'$ is convex in our case; thus, we obtain

$$\phi'(V^*_B(s^*)) < \phi' \left( \sum_{S_k \in B} \alpha(S_k)V_{S_k}(s) \right) < \sum_{S_k \in B} \alpha(S_k) \phi'(V_{S_k}(s)),$$
where the first \(<\) uses \([4]\) and \(\phi'\) is decreasing, and the second \(<\) follows from Jensen’s inequality.

Finally,

\[
\frac{\lambda_B}{\lambda_\pi} = \frac{\phi'(V^*_B(s^*))}{\sum_{S_k \in B} \alpha(S_k) \phi'(V^*_S(s))} \ < \ 1.
\]

The rest follows from Lemma 12.

**Proof of Lemma 5.** Take any \(s_i \in S_k \in \rho \setminus B\), \(c_i = c^*_i\) and the ratio of the FOCs for \(\pi\)- and \(\rho\)-aggregation problems is

\[
\frac{\lambda_\rho}{\lambda_\pi} = \frac{\phi'(V_{S_k}(\tilde{s})) u'(\tilde{c}_i)}{\phi'(V_{S_k}(s)) u'(c_i)}.
\]

By Taylor approximation,

\[
\frac{\lambda_\rho}{\lambda_\pi} \approx \frac{\phi'(V_{S_k}(\tilde{s}))}{\phi'(V_{S_k}(s))} [1 - A_u(c_i) \Delta \tilde{c}_i] = \frac{\phi'(V_{S_k}(\tilde{s}))}{\phi'(V_{S_k}(s))} [1 - \gamma \Delta \tilde{c}_i].
\]

Clearly, \(\Delta \tilde{c}_i\) remains constant for all the states in \(S_k\).

The proof for \(s_i \in B\) goes by analogy, except that we take the ratio of FOCs for \(B\)- and \(\rho\)-aggregation problems, implying instead \(\frac{\lambda_\rho}{\lambda_B}\) on the left-hand side. It follows that \(\Delta \tilde{c}_i\) is constant on event \(B\).

**Lemma 13.** \(\Delta \tilde{V}_{S_k} \approx \Delta \tilde{c}_{S_k} (1 - \gamma V_{S_k})\).

**Proof.** By definition,

\[
V_{S_k}(s^*) = \sum_{s_i \in S_k} P(s_i|S_k) u(c^*_i) = \sum_{s_i \in S_k} P(s_i|S_k) \frac{1}{1 - e^{-\gamma c^*_i}}.
\]
Hence,

\[ \Delta \tilde{V}_{S_k} = \sum_{s_i \in S_k} P(s_i | S_k) \frac{1}{\gamma} (1 - e^{-\gamma c_i}) - \sum_{s_i \in S_k} P(s_i | S_k) \frac{1}{\gamma} (1 - e^{-\gamma \tilde{c}_i}) \]

\[ = \sum_{s_i \in S_k} P(s_i | S_k) \frac{1}{\gamma} (e^{-\gamma \tilde{c}_i} - e^{-\gamma c_i}) \]

\[ \approx \sum_{s_i \in S_k} P(s_i | S_k) \frac{1}{\gamma} (e^{-\gamma c_i} \gamma \Delta \tilde{c}_i) \quad (5) \]

\[ = \left[ \sum_{s_i \in S_k} P(s_i | S_k) \frac{1}{\gamma} (e^{-\gamma c_i}) \right] \gamma \Delta \tilde{c}_{S_k}, \quad (6) \]

where (5) uses Taylor approximation and (6) uses Lemma 5.

Thus,

\[ \Delta \tilde{c}_{S_k} - \Delta \tilde{V}_{S_k} = \left[ \sum_{s_i \in S_k} P(s_i | S_k) \frac{1}{\gamma} (1 - e^{-\gamma c_i}) \right] \gamma \Delta \tilde{c}_{S_k} = V_{S_k}(s^*) \gamma \Delta \tilde{c}_{S_k} \]

\[ \Delta \tilde{V}_{S_k} = (1 - \gamma V_{S_k}(s^*)) \Delta \tilde{c}_{S_k}. \]

**Proof of Lemma 6.** For any \( S_k \subseteq B^c \), no consumption change occurs under the aggregation effect, and hence \( V_{S_k}^* = V_{S_k} \). The ratio of FOC from PDS \( \pi \) to PDS \( \rho \) implies

\[ \frac{\phi'(V_{S_k}) u'(\tilde{c}_i)}{\phi'(V_{S_k}) u'(c_i)} = \frac{\lambda_\rho}{\lambda_\pi}. \]

Substituting the CARA functional forms of \( u \) implies

\[ \left( \frac{\phi'(V_{S_k})}{\phi'(V_{S_k})} \right) \frac{e^{-\gamma \tilde{c}_i}}{e^{-\gamma c_i}} = \frac{\lambda_\rho}{\lambda_\pi} \]

\[ \left( 1 + \frac{\phi''(V_{S_k}) \Delta \tilde{V}_{S_k}}{\phi'(V_{S_k})} \right) e^{-\gamma \Delta \tilde{c}_i} \approx \frac{\lambda_\rho}{\lambda_\pi} \quad (7) \]

\[ \left( 1 - A \phi(V_{S_k}) \Delta \tilde{V}_{S_k} \right) e^{-\gamma \Delta \tilde{c}_{S_k}} \approx \frac{\lambda_\rho}{\lambda_\pi}, \quad (8) \]

where (7) follows from Taylor approximation and (8) follows from Lemma 5. And substitut-
ing the expression from Lemma 13 yields

\[ [1 - A_\phi(V_{S_k})(1 - \gamma V_{S_k})\Delta \tilde{c}_{S_k}] e^{-\gamma \Delta \tilde{c}_{S_k}} \approx \frac{\lambda_B}{\lambda_\pi}. \]  

(9)

Note for a given value of \( V_{S_k} \), \( \Delta \tilde{c}_{S_k} \) is determined by \( V_{S_k} \) via equation (9), whereas \( V_{S_k} \) does not depend on \( \Delta \tilde{c}_{S_k} \). For simplicity, use notation \( x \) for \( V_{S_k} \), \( y \) for \( \Delta \tilde{c}_{S_k} \), and \( F(x, y) \) for the left-hand side of equation (9). Let \( y(x) \) be the implicit function that determines the value of \( y \) for each \( x \) that solves equation (9), which is \( F(x, y(x)) = \frac{\lambda_B}{\lambda_\pi} \). As the expression below shows, by the implicit function theorem, because the regularity condition \( \partial F/\partial y \neq 0 \) clearly holds, the function \( y(x) \) is differentiable and

\[
\frac{dy(x)}{dx} = -\frac{\partial F/\partial x}{\partial F/\partial y} = -\frac{-A'_\phi(x)(1 - \gamma x)ye^{-\gamma y} + A_\phi(x)\gamma ye^{-\gamma y}}{e^{-\gamma y}[-\gamma(1 - A_\phi(x)(1 - \gamma x)y) - A_\phi(x)(1 - \gamma x)] - A'_\phi(x)(1 - \gamma x) + A_\phi(x)\gamma}.
\]

Note \( \gamma > 0 \), \( y = \Delta \tilde{c}_{S_k} > 0 \) for \( S_k \in \rho \setminus B \) (by Lemma 4 and Lemma 12), and \( 1 - \gamma x > 0 \) as \( x = V_{S_k} \in (0, \frac{1}{\gamma}) \). By equation (9), \( 1 - A_\phi(x)(1 - \gamma x)y \approx (\lambda_B/\lambda_\pi) \cdot e^{\gamma y} > 0 \). Also, \( A_\phi > 0 \) and \( A'_\phi \leq 0 \) (by Assumption 2). Hence, the above derivative is positive; that is, \( \Delta \tilde{c}_{S_k} \) is increasing in \( V_{S_k} \).

\[ \square \]

Proof of Proposition 2. \( r_{S_k} \) is increasing in \( k \). Because \( \lambda_B < \lambda_\pi \), we have (i) \( \Delta I_B < 0 \), and hence \( \Delta \tilde{c}_B < 0 \) for all \( s_i \in B \); (ii) \( \Delta I_{B^c} > 0 \), and hence \( \Delta \tilde{c}_{S_k} > 0 \) for all \( S_k \subseteq B^c \). Hence, \( \Delta \tilde{s} > 0 \) and \( \Delta \tilde{d}_B = \Delta \tilde{s} - \Delta \tilde{c}_B > 0 \) for all \( s_i \in B \).

And for all \( S_k \subseteq B^c \), the following statements are equivalent: (i) \( r_{S_k} \geq r_1 \); (ii) \( \tilde{V}_{S_k} \leq \tilde{V}_{s_1} \); (iii) \( \Delta \tilde{c}_{S_k} \leq \Delta \tilde{s} \); and (iv) \( \Delta \tilde{d}_{S_k} \geq 0 \).

Again, we can define \( S_+ = \{ S_k \in \rho : \Delta \tilde{d}_{S_k} \geq 0 \} \) and \( S_- = \{ S_k \in \rho : \Delta \tilde{d}_{S_k} < 0 \} \). Because
\( r_B \geq r_1, B \in S_+ \). Hence, \( r_B \geq \min_{s_k} r_{s_k} \geq r_1 > \max_{s_k} r_{s_k} \). Thus,

\[
\Delta \tilde{L} = \sum_{s_k \in \rho} p_{s_k} \Delta \tilde{d}_{s_k} \left( 1 - \frac{1}{r_{s_k}} \right)
= \sum_{s_k \in S_+} p_{s_k} \Delta \tilde{d}_{s_k} \left( 1 - \frac{1}{r_{s_k}} \right) + \sum_{s_k \in S_-} p_{s_k} \Delta \tilde{d}_{s_k} \left( 1 - \frac{1}{r_{s_k}} \right)
> \left( \sum_{s_k \in S_+} p_{s_k} \Delta \tilde{d}_{s_k} \right) \left( 1 - \frac{1}{\min_{s_k} r_{s_k}} \right) + \left( \sum_{s_k \in S_-} p_{s_k} \Delta \tilde{d}_{s_k} \right) \left( 1 - \frac{1}{\max_{s_k} r_{s_k}} \right)
\geq \left( 1 - \frac{1}{\min_{s_k} r_{s_k}} \right) \sum_{s_k \in \rho} p_{s_k} \Delta \tilde{d}_{s_k} = \left( 1 - \frac{1}{\min_{s_k} r_{s_k}} \right) \Delta \tilde{s} \geq \left( 1 - \frac{1}{r_1} \right) \Delta \tilde{s} \geq 0,
\]

where the last \( \geq \) follows from \( \Delta \tilde{s} > 0 \) and \( r_1 \geq 1 \).

\( \square \)

### A.3 Generalization

#### A.3.1 Aggregation effect

**Proof of Lemma 7.** First, note consumption inside each event is monotone, due to the FOCs

\[
\frac{u'(c_i)}{u'(c_j)} = \frac{r_i}{r_j}.
\]

Because \( u \) is concave, \( r_i < r_j \) implies \( c_i > c_j \) for \( s_i, s_j \in S_k \).

Now consider concave \( \phi \). Suppose not and \( V_{S_k} \leq V_{S_l} \). By concavity of \( \phi \) and by monotonicity \( \frac{r_i}{r_j} < 1 \), we have

\[
\frac{\phi'(V_{S_k})}{\phi'(V_{S_l})} \geq 1 \quad \Rightarrow \quad \frac{u'(c_i)}{u'(c_j)} < 1 \quad \text{for all} \quad s_i \in S_k, s_j \in S_l
\]

\[
\Rightarrow \quad c_i > c_j \quad \text{for all} \quad s_i \in S_k, s_j \in S_l.
\]

Suppose not and \( V_{S_k} \leq V_{S_l} \). Because \( \phi(u(\cdot)) \) is concave,

\[
\frac{\phi'(V_{S_k}(s)) u'(c_i)}{\phi'(V_{S_l}(s)) u'(c_j)} = \frac{r_i}{r_j} < 1 \quad \Rightarrow \quad c_i > c_j \quad \text{for all} \quad s_i \in S_k, s_j \in S_l.
\]

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Hence,

\[ V_{S_k} = \sum_{s_i \in S_k} u(c_i) P(s_i | S_k) \]
\[ \geq \min_{s_i \in S_k} u(c_i) > \max_{s_j \in S_l} u(c_j) \]
\[ \geq \sum_{s_j \in S_l} u(c_j) P(s_j | S_l) = V_{S_l}, \]

which leads to a contradiction.

For the consumption sequence to be monotonically decreasing, note

\[ (\phi(u(x)))' = \phi'(u(x))u'(x) \]
\[ (\phi(u(x)))'' = \phi''(u(x))(u'(x))^2 + \phi'(u(x))u''(x) \]
\[ -\frac{(\phi(u(x)))''}{(\phi(u(x))'} = -\frac{\phi''(u(x))u'(x)}{\phi'(u(x))} - u''(x) \]

Hence,

\[ A_{\phi\circ u}(x) - A_{u}(x) = A_{\phi}(u(x))u'(x). \]

The FOCs are

\[ [1 - A_{\phi}(V_{k+1})(V_k - V_{k+1})] \min_{s_j \in S_{k+1}} r_j = \frac{u'(c_{\min_{s_i \in S_k} S_{k+1}})}{u'(c_{\max_{s_i \in S_k} S_{k+1}})} \]
\[ \max_{s_i \in S_k} r_i, \]

(10)

where \( \max S_k \) and \( \min S_{k+1} \) are the last and the first state in \( S_k \) and \( S_{k+1} \), respectively. It suffices to show the left-hand side is greater than 1 for all \( S_k \in \pi \).

\[ V_k - V_{k+1} = u(c_{S_k}) - u(c_{S_{k+1}}) \leq u'(c_{S_{k+1}}) [c_{S_k} - c_{S_{k+1}}] \leq u'(c_{S_{k+1}}) \left[ \max_{s_i \in S_k} c_i - \min_{s_j \in S_{k+1}} c_j \right], \]

where \( c_{S_k} \) and \( c_{S_{k+1}} \) are the certainty equivalents to consumptions in \( S_k \) and \( S_{k+1} \). Let \( A_{\phi}(V_{k+1}) = A_{\phi}(u(c_{S_{k+1}})) \); then Assumption 4 implies the left-hand side of equation (10) is greater than 1.

**Proof of Proposition 3.** By the monotonicity assumption, \( r_i = \frac{p_i}{P(s_i)} \) is increasing in \( i \);
thus, \( \frac{P_i}{A_u(c_i)} \) is also increasing in \( i \). The last statement implies the normalized sequence

\[
\frac{P_i}{\sum_{s_i \in B} P(s_i)} \frac{A_u(c_i)}{\sum_{s_i \in B} A_u(c_i)} \]

is increasing in \( i \). Finally, by monotone aggregation of the states, we get that

\[
\alpha(S_k) = \frac{\sum_{s_i \in S_k} p_i A_u(c_i)}{\sum_{s_i \in S_k} P(s_i)} \frac{A_u(c_i)}{\sum_{s_i \in B} A_u(c_i)}
\]

is increasing in \( k \). Now note that \( V_{S_k}(s) \) is decreasing in \( k \) by Lemma 7 and \( \phi \) is concave, so \( \phi'(V_{S_k}(s)) \) is increasing in \( k \), implying

\[
\sum_{S_k \in B} \alpha(S_k) \phi'(V_{S_k}(s)) > \sum_{S_k \in B} \beta(S_k) \phi'(V_{S_k}(s)).
\]

By Lemma 9, the aggregation effect on losses is

\[
\Delta L^* = \sum_{s_i \in B} \frac{P(s_i)}{A_u(c_i)} \left( 1 - \frac{\beta(S_k) \phi'(V_{S_k}(s))}{\alpha(S_k) \phi'(V_{S_k}(s))} \right) > 0.
\]

By Lemma 7, \( \Gamma_{S_k} \) is decreasing in \( k \). By Assumption 4, \( A_u(c_i) \) is non-decreasing in \( i \). Lemma 8 implies \( \Delta c_i^* \) is non-increasing.

\[\square\]

### A.3.2 Event-smoothing effect

**Lemma 14.** Given \( u'(x) > 0 \), Assumption 6 is equivalent to \( g(x) = \frac{(u'(x))^2}{u''(x)} \) being non-decreasing.

**Proof.** Given \( u'(x) > 0 \), note

\[
g'(x) = 2u'(x) - \frac{(u'(x))^2 u'''(x)}{(u''(x))^2} \geq 0 \iff 2 \geq \frac{(u'(x))^3 u'''(x)}{(u''(x))^3} \iff 2A_u(x) \geq P_u(x).
\]
Also,
\[ 1 \geq T_u(x)' = \left( \frac{-u'(x)}{u''(x)} \right)' = \frac{-(u''(x))^2 + u'''(x)u'(x)}{(u''(x))^2}, \]
which holds if and only if
\[ (u''(x))^2 \geq -(u''(x))^2 + u'''(x)u'(x) \iff 2(u''(x))^2 \geq u'''(x)u'(x) \iff 2A_u(x) \geq P_u(x). \]

Lemma 15. If \( u \) and \( \phi \) are concave and three-times differentiable, and Assumptions 3, 4, and 6 hold, \( \frac{r_j}{A_u(c_j)} \) is non-decreasing in \( j \).

Proof. Consider state \( s_j \in S_k \) and its FOC:
\[ \phi'(V_{S_k}(s)) = \lambda_{\pi} r_i \Rightarrow \phi'(V_{S_k}(s)) \frac{u'(c_i)}{A_u(c_i)} = \lambda_{\pi} \frac{r_i}{A_u(c_i)}. \]

First, note \( \phi \) is concave and \( V_{S_k}(s) \) is decreasing in \( k \) by Lemma 7, so \( \phi'(V_{S_k}(s)) \) is increasing in \( k \).

Second, note that by Lemma 14, Assumption 6 is equivalent to \( g(x) = \frac{(u'(x))^2}{u''(x)} \) being non-decreasing. In addition, \( \frac{u'(x)}{A_u(x)} = -\frac{(u'(x))^2}{u''(x)} \), implying \( \frac{u'(x)}{A_u(x)} \) is a non-increasing function; hence, \( \frac{u'(c_i)}{A_u(c_i)} \) is non-decreasing in \( i \) because \( c_i \) decreases in \( i \) by Lemma 7 and the result follows.

Lemma 16. If \( u \) is concave and three-times differentiable and Assumptions 3, 4 and 6 hold, \( \lambda_\rho < \lambda_\pi \).

Proof. DARA \( \phi \) implies \( \phi''' > 0 \). By Lemma 15, Assumptions 3, 4 and 6 imply \( \frac{r_j}{A_u(c_j)} \) is non-decreasing in \( j \).

For all \( s_j \in S_i \in B \), dividing the FOCs from the problem with \( \pi \) and the aggregation problem on \( B \) implies
\[ \frac{\phi'(V_B(s^*))u'(c_j^*)}{\phi'(V_{S_i}(s))u'(c_j)} = \frac{\lambda_B}{\lambda_{\pi}}. \]

By construction,
\[ V_B(s^*) = \sum_{s_i \in B} P(s_i|B)u(c_i^*) \geq V_B(s) = \sum_{s_i \in B} P(s_i|B)u(c_i). \]
By first-order Taylor approximation of \( u'(c_j^*) \) at \( c_j \),

\[
\frac{\phi'(V_B(s^*))}{\phi'(V_S_i(s))} [1 - A_u(c_j) \Delta c_j^*] = \frac{\lambda_B}{\lambda_{\pi}}.
\]

That is,

\[
\Delta c_j^* = \frac{1}{A_u(c_j)} \left( 1 - \frac{\lambda_B}{\lambda_{\pi}} \cdot \frac{\phi'(V_S_i(s))}{\phi'(V_B(s^*))} \right).
\]

Because total expenditure on \( B \) remains unchanged, we have

\[
0 = \sum_{s_j \in B} p_j \Delta c_j^* = \sum_{s_j \in B} \frac{p_j}{A_u(c_j)} \left( 1 - \frac{\lambda_B}{\lambda_{\pi}} \cdot \frac{\phi'(V_S_i(s))}{\phi'(V_B(s^*))} \right),
\]

\[
\frac{\lambda_B}{\lambda_{\pi}} = \frac{\sum_{s_j \in B} \frac{p_j}{A_u(c_j)} \phi'(V_S_i(s))}{\sum_{S_i \subseteq B} \frac{1}{\phi'(V_B(s^*))} \left( \sum_{s_j \in S_i} \frac{p_j}{A_u(c_j)} \right)} = \frac{\phi'(V_B(s^*))}{\sum_{S_i \subseteq B} \alpha(S_i) \phi'(V_S_i(s))}.
\]

For each \( S_i \subseteq B, S_i \in \pi \), observe that

\[
\alpha(S_i) = \sum_{s_j \in S_i} \frac{p_j}{A_u(c_j)} \frac{r_j}{A_u(c_j)} = \frac{\sum_{s_j \in S_i} \frac{r_j}{A_u(c_j)} P(s_j)}{\sum_{s_j \in B} \frac{r_j}{A_u(c_j)} P(s_j)}
\]

is a conditional probability on event partition \( \pi_B = \{S_k, \ldots, S_l\} \), transformed from probability \( P(\cdot|B) \) with Radon-Nikodym derivative \( \frac{r_j}{A_u(c_j)} \). Because \( \frac{r_j}{A_u(c_j)} \) is non-decreasing, \( \frac{\alpha(S_k)}{P(S_k|B)} \) is non-decreasing in the index \( k \). By Lemma 7, \( V_{S_k} \) is decreasing in \( k \). Because \( \phi'(\cdot) \) is decreasing and convex,

\[
\sum_{i=k}^{l} \alpha(S_i) \phi'(V_S_i(s)) \geq \sum_{i=k}^{l} P(S_i|B) \phi'(V_S_i(s)) > \phi'(V_B(s)) \geq \phi'(V_B(s^*)).
\]

Hence, \( \lambda_B < \lambda_{\pi} \). By lemma 12, \( \lambda_{\rho} < \lambda_{\pi} \Leftrightarrow \lambda_B < \lambda_{\pi} \) and the result follows.

\[\square\]

**Lemma 17** (Event-smoothing effect). If \( u \) and \( \phi \) are concave and three-times differentiable, then for any \( s_i \in S_k \in \rho \), the event-smoothing effect can be calculated as \( \Delta \tilde{c}_i = \frac{\tilde{r}_{S_k}}{A_u(c_i^*)} \), where
\[
\begin{align*}
\lambda_{\rho} & = (1 - A_\phi(V_S(s))\tilde{\Gamma}_S E_S)(1 - \tilde{\Gamma}_S) & \text{for any } S_k \in \rho \setminus B \\
\lambda_{\rho} & = \lambda_B(1 - A_\phi(V_B(s^*))\tilde{\Gamma}_B E_B)(1 - \tilde{\Gamma}_B) \\
0 & = \sum_{S_k \in \rho} \alpha^*(S_k)\tilde{\Gamma}_S,
\end{align*}
\]

where
\[
\frac{\lambda_B}{\lambda_\pi} = \frac{\phi'(V_B(s^*))}{\sum_{S_k \in B} \alpha(S_k)\phi'(V_S(s))}.
\]

Proof. Consider first \( s_i \in S_k \in \rho \setminus B \), the ratio of the FOCs for \( \pi - \) and \( \rho - \)aggregation problems is
\[
\frac{\lambda_{\rho}}{\lambda_\pi} = \frac{\phi'(V_S(\tilde{s})) u'(\tilde{c}_i)}{\phi'(V_S(s)) u'(c_i)},
\]

however, note
\[
\frac{\phi'(V_S(\tilde{s}))}{\phi'(V_S(s))} = 1 + \frac{\phi''(V_S(s))}{\phi'(V_S(s))} \Delta \tilde{V}_S(s) = 1 - A_\phi(V_S(s)) \Delta \tilde{V}_S(s)
\]
\[
\frac{u'(\tilde{c}_i)}{u'(c_i)} = 1 + \frac{u''(c_i)}{u'(c_i)} \Delta \tilde{c}_i = 1 - A_u(c_i) \Delta \tilde{c}_i,
\]

implying
\[
\frac{\lambda_{\rho}}{\lambda_\pi} = (1 - A_\phi(V_S(s)) \Delta \tilde{V}_S(s))(1 - A_u(c_i) \Delta \tilde{c}_i).
\]

Denote \( \tilde{\Gamma}_{S_k} = A_u(c_i) \Delta \tilde{c}_i = 1 - \frac{\lambda_{\rho} \phi'(V_S(\tilde{s}))}{\lambda_\pi \phi'(V_S(s))} \). Then, \( \Delta \tilde{c}_i = \frac{\tilde{\Gamma}_{S_k}}{A_u(c_i)} = \frac{\tilde{\Gamma}_{S_k}}{A_u(c_i)} \) since the events not in \( B \) are not affected by the aggregation effect. Also, note
\[
\Delta \tilde{V}_S(s) = \sum_{s_i \in S_k} P(s_i|S_k)(u(\tilde{c}_i) - u(c_i)) = \sum_{s_i \in S_k} P(s_i|S_k)u'(c_i) \Delta \tilde{c}_i = \tilde{\Gamma}_S E_S.
\]

Hence,
\[
\frac{\lambda_{\rho}}{\lambda_\pi} = (1 - A_\phi(V_S(s)) \tilde{\Gamma}_S E_S)(1 - \tilde{\Gamma}_S).
\]

And so we have obtained a quadratic equation for \( \tilde{\Gamma}_{S_k} \).

The proof for event \( B \) goes by analogy, with the exception that we should take the ratio of
the FOCs from the $\rho$–aggregation problem and $B$–intermediate bundle problem, implying $\frac{\lambda_u}{\lambda_B}$ would appear instead of $\frac{\lambda_u}{\lambda_B}$. However, note $\frac{\lambda_u}{\lambda_B} = \frac{\lambda_B}{\lambda_B}$. And the second equation follows.

Finally, note the total income does not change, so the sum of the change in income must be zero:

$$\sum_{s_i \in \Omega} p_i \Delta \tilde{c}_i = \sum_{s_i \in \Omega} \frac{p_i}{A_u(c^*_i)} \tilde{\Gamma}_{S_k} = \sum_{s_i \in S_k} \frac{p_i}{A_u(c^*_i)} \sum_{S_k \in \rho} \tilde{\Gamma}_{S_k} = \sum_{S_k \in \rho} \alpha^*(S_k) \tilde{\Gamma}_{S_k} = 0.$$

Lemma 18. If $u$ is concave and three-times differentiable, and Assumptions 2–4 and 6 hold, $E_{S_k} A_{\phi}(V_{S_k}(s))$ does not decrease in $k$.

Proof. First, consider

$$E_{S_k} = \sum_{s_i \in S_k} P(s_i|S_k) \frac{u'(c_i)}{A_u(c_i)} = - \sum_{s_i \in S_k} P(s_i|S_k) \frac{(u'(c_i))^2}{u''(c_i)}.$$

By Lemma 7, $c_i$ decreases in $i$, and by Lemma 14, $(\frac{u'(x)}{u''(x)})^2$ is non-increasing; hence, $(\frac{u'(c_i))^2}{u''(c_i)}$ does not increase in $i$. Due to monotonic aggregation of the states, we get that $E_{S_k}$ does not decrease in $k$.

Now consider $A_{\phi}(V_{S_k}(s))$. $V_{S_k}(s)$ decreases in $k$ by Lemma 7. In addition, $\phi$ is DARA, implying $A_{\phi}(\cdot)$ is a non-increasing function. Hence, $A_{\phi}(V_{S_k}(s))$ does not decrease in $k$. Then, the result follows. \qed

Lemma 19. If $u$ and $\phi$ are concave and three-times differentiable, $\tilde{\Gamma}_{S_k} > (\phi)0$ and $A_{\phi}(V_{S_k}(s))E_{S_k}$ does not decrease in $k$ for any $S_k \in \rho \setminus B$, then $\tilde{\Gamma}_{S_k}$ does not increase (decrease) in $k$.

Proof. To simplify notation of this proof, we denote $A_{\phi}(V_{S_k}(s))E_{S_k} = a_k$ inside this proof only. First, consider the quadratic equation from Lemma 17:

$$\frac{\lambda_u}{\lambda_B} = (1 - a_k \tilde{\Gamma}_{S_k})(1 - \tilde{\Gamma}_{S_k})$$

for any $S_k \in \rho \setminus B$,

which can be rewritten as

$$a_k \tilde{\Gamma}_{S_k}^2 - (a_k + 1) \tilde{\Gamma}_{S_k} + 1 - \frac{\lambda_u}{\lambda_B} = 0.$$
Now we take the derivative of the above equation with respect to $a_k$ and get the following:

$$\dot{\Gamma}_{S_k}^2 + 2a_k \dot{\Gamma}_{S_k} \frac{d\Gamma_{S_k}}{da_k} - \dot{\Gamma}_{S_k} - (a_k + 1) \frac{d\Gamma_{S_k}}{da_k} = 0$$

$$\Rightarrow \frac{d\Gamma_{S_k}}{da_k} = \frac{\dot{\Gamma}_{S_k}(1 - \dot{\Gamma}_{S_k})}{2a_k \dot{\Gamma}_{S_k} - a_k - 1}.$$

We are interested in the sign of $\frac{d\Gamma_{S_k}}{da_k}$.

The actual solutions to the equation are

$$\dot{\Gamma}_{S_k} = \frac{a_k + 1}{2a_k} \pm 0.5 \sqrt{\left(1 + \frac{1}{a_k}\right)^2 - \frac{4}{a_k} \left(1 - \frac{\lambda_{\rho}}{\lambda_{\pi}}\right)}.$$

Note that when $\frac{\lambda_{\pi}}{\lambda_{\rho}} = 1$, $\dot{\Gamma}_{S_k} = 0$, because the event-smoothing effect is 0 in this case; hence we can throw away the solution with “+”. Thus, $\dot{\Gamma}_{S_k} \leq \frac{a_k + 1}{2a_k}$.

In addition, $(1 - a_k \dot{\Gamma}_{S_k})(1 - \dot{\Gamma}_{S_k}) > 0$, implying both elements $(1 - a_k \dot{\Gamma}_{S_k})$ and $(1 - \dot{\Gamma}_{S_k})$ must be of the same sign. The greater of the roots of the equation produces negative elements, whereas the smaller produces positive elements, because $1 - \dot{\Gamma}_{S_k}$ is smaller when $\dot{\Gamma}_{S_k}$ is greater. Thus, $\dot{\Gamma}_{S_k} < 1$ because we are dealing with the smallest of the two roots. Hence, we obtain $\text{sgn}\left(\frac{d\Gamma_{S_k}}{da_k}\right) = -\text{sgn}\dot{\Gamma}_{S_k}$ and the result follows.

Proof of Proposition 4. Because Assumptions [2] [9] hold, by Lemma 16, $\lambda_{\pi} > \lambda_{\rho}$. Then, Lemma 12 implies $\Delta I_{B^c} > 0$, $\Delta \tilde{s} > 0$, and $\Delta \tilde{c}_i > 0$ for all $s_i \in S_k \in \rho \setminus (B \cup \{s_1\})$. By Lemma 18, $E_{S_k} A_{\phi}(V_{S_k}(s))$ does not decrease in $k$ for all $S_k \in \rho \setminus (B \cup \{s_1\})$.

For all $s_i \notin B \cup \{s_1\}$, Lemma 17 implies $\Delta \tilde{c}_i = \frac{\dot{\Gamma}_{S_k}}{A_{\phi}(c_i)}$. Hence, for all $S_k \in \rho \setminus (B \cup \{s_1\})$, $\dot{\Gamma}_{S_k} > 0$. By Lemma 19, we have that $\dot{\Gamma}_{S_k}$ does not increase in $k$. By Assumption 5, $u$ is DARA, and $c_i$ is decreasing in $i$, $\Delta \tilde{c}_i$ is non-increasing in $i$. Therefore, we have that $\Delta \tilde{c}_i = \Delta \tilde{s} - \Delta \tilde{c}_i$ is non-decreasing in $i$ for all $s_i \notin B \cup \{s_1\}$.

Define $S_+ = \{S_k \in \rho : \Delta \tilde{d}_i \geq 0, \forall s_i \in S_k\}$ and $S_- = \{S_l \in \rho : \Delta \tilde{d}_j < 0, \forall s_j \in S_l\}$. By Lemma 19, $S_-$ contains all the events $S_k \in \rho \setminus B$ such that $r_i < r_1$ for all $s_i \in S_k$. And $S_+$ contains all the events $S_l \in \rho \setminus B$ such that $r_1 < r_j$ for all $s_j \in S_l$. Because $\Delta \tilde{d}_i > 0$ for all $s_i \in B, B \in S_+$. Then, the condition $r_1 < \min_{s_j \in S_{i+1}} r_j$ implies $S_+$ contains event $B$ and all
the events $S_k \in \rho$ after $B$. Therefore, $\min_{S_+} > \max_{S_-}$. Thus,

$$\Delta \tilde{L} = \sum_{s_i \in S} p_i \Delta \tilde{d}_i \left(1 - \frac{1}{r_i}\right)$$

$$= \sum_{S_k \in S_+} \sum_{s_i \in S_k} p_i \Delta \tilde{d}_i \left(1 - \frac{1}{r_i}\right) + \sum_{S_l \in S_-} \sum_{s_j \in S_l} p_j \Delta \tilde{d}_j \left(1 - \frac{1}{r_j}\right)$$

$$> \left( \sum_{S_k \in S_+} \sum_{s_i \in S_k} p_i \Delta \tilde{d}_i \right) \left(1 - \frac{1}{\min_{S_+} r_i}\right) + \left( \sum_{S_l \in S_-} \sum_{s_j \in S_l} p_j \Delta \tilde{d}_j \right) \left(1 - \frac{1}{\max_{S_-} r_j}\right)$$

$$\geq \left(1 - \frac{1}{\min_{S_+} r_i}\right) \sum_{s_i \in S} p_i \Delta \tilde{d}_i = \left(1 - \frac{1}{\min_{S_+} r_i}\right) \Delta \tilde{s} \geq 0,$$

where the last $\geq$ follows from $\min_{S_+} r_i \geq r_1 \geq 1$ and $\Delta \tilde{s} > 0$.

Proof of Lemma 10.

$$\Delta \tilde{L} = \sum_{s_i \in \Omega \setminus \{s_1\}} (p_i - P(s_i)) (\Delta \tilde{s} - \Delta \tilde{c}_i) = -(p_1 - P(s_1)) \Delta \tilde{s} - \sum_{s_i \in \Omega \setminus \{s_1\}} (p_i - P(s_i)) \Delta \tilde{c}_i$$

$$= - \sum_{s_i \in \Omega} p_i \Delta \tilde{c}_i + \sum_{s_i \in \Omega} P(s_i) \Delta \tilde{c}_i = \sum_{s_i \in \Omega} P(s_i) \Delta \tilde{c}_i = \sum_{S_k \in \rho} \tilde{\Gamma}_{S_k} \sum_{s_i \in S_k} \frac{P(s_i)}{A_u(c^*_i)}$$

$$= \left( \sum_{s_i \in \Omega} \frac{P(s_i)}{A_u(c^*_i)} \right) \sum_{S_k \in \rho} \beta^*(S_k) \tilde{\Gamma}_{S_k}.$$

References


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16In other words, the sequence $\{\Delta \tilde{d}_i : i \geq 2\}$ crosses zero from below exactly once.


